

THE RANGE CHARACTERIZATIONS OF THE TOTALLY
GEODESIC RADON TRANSFORM ON THE REAL
HYPERBOLIC SPACE

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0. Introduction.....	149
1. Preliminaries	154
2. The properties of differential operators	159
3. The definition of L^p -type rapidly decreasing function spaces on G/K or $G/H_0^{(k)}$ and the definition of the totally geodesic Radon transform.....	163
4. The Poisson transform and the explicit computation of spherical functions.....	166
5. The Payley-Wiener-type theorem for $S^p(G/K)$ and the asymptotic expansion of rapidly decreasing functions.....	172
6. The inclusion theorem for $\mathfrak{F}^{(k)}(S^p(G/H_0^{(k)}))$	178
7. The relation between the Radon transform and the Fourier transform.....	181
8. The asymptotic property of R	183
9. The injectivity theorem of the Fourier transform on $G/H_0^{(k)}$	184
10. The characterizations of $R(S^p(G/K))$	192
11. The characterizations of $R(C_0^\infty(G/K))$	193

0. Introduction. Let G be a real Lie group, K and H be its closed unimodular subgroups, and $\Gamma(G/K)$ be a rapidly decreasing function space in $C^\infty(G/K)$. If

$$(R.f)(g.H) = \int_H f(gh.K) dh$$

converges for all $g \in G$ and $R.f \in C^\infty(G/H)$ for all $f \in \Gamma(G/K)$ where dh is the invariant measure on H , then we can define the integral transform

$$R: \Gamma(G/K) \rightarrow C^\infty(G/H),$$

and R is a G -module homomorphism. Thus the range space $R(\Gamma(G/K))$ is annihilated by $\text{Ann}_{U(\mathfrak{g})} C^\infty(G/K)$, namely, the annihilator ideal of the left $U(\mathfrak{g})$ -module $C^\infty(G/K)$ where \mathfrak{g} is the Lie algebra of G and $U(\mathfrak{g})$ is the universal enveloping algebra of \mathfrak{g} . This implies that all the functions in the range space satisfy any linear partial differential equation on G/H constructed from the left

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