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THE RANGE CHARACTERIZATIONS OF THE TOTALLY GEODESIC RADON TRANSFORM ON THE REAL HYPERBOLIC SPACE

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0. Introduction. Let G be a real Lie group, K and H be its closed unimodular subgroups, and $\Gamma(G/K)$ be a rapidly decreasing function space in $C^\infty(G/K)$. If

$$(R.f)(g.H) = \int_H f(gh.K)\,dh$$

converges for all $g \in G$ and $R.f \in C^\infty(G/H)$ for all $f \in \Gamma(G/K)$ where dh is the invariant measure on H, then we can define the integral transform

$$R\colon \Gamma(G/K) \to C^\infty(G/H),$$

and R is a G-module homomorphism. Thus the range space $R(\Gamma(G/K))$ is annihilated by $\mathrm{Ann}_{U(\mathfrak{g})} C^\infty(G/K)$, namely, the annihilator ideal of the left $U(\mathfrak{g})$-module $C^\infty(G/K)$ where $\mathfrak{g}$ is the Lie algebra of G and $U(\mathfrak{g})$ is the universal enveloping algebra of $\mathfrak{g}$. This implies that all the functions in the range space satisfy any linear partial differential equation on G/H constructed from the left

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