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UNIRATIONALITY OF FANO VARIETIES

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1. Introduction. This note is an extension of the paper [HMP]. The main theorem of [HMP] states that a relatively smooth hypersurface (i.e., a hypersurface whose singular locus has sufficiently large codimension with respect to its degree) is unirational. A mild modification can generalize this to complete intersections. As an application, we will show that the Fano variety of a relatively smooth hypersurface is also unirational. Precisely, we have the following theorem.

THEOREM 1.1 (Unirationality of Fano varieties). *Let X be a degree d hypersurface over an algebraically closed field. The Fano variety $F_k(X)$ of X is unirational if the codimension of singular locus of X is sufficiently large with respect to both d and k. Here by "sufficiently large," we mean for any integers d and k, there exists a number $N(d,k)$ such that $F_k(X)$ is unirational if the codimension of singular locus of X is greater than $N(d,k)$.*

Remarks. 1. Throughout the note, we work exclusively on the fields of characteristic 0.

2. The Fano variety $F_k(X)$ of a projective variety $X \subset \mathbb{P}^n$ is defined to be the subvariety of the Grassmannian $\mathbb{G}(k,n)$ consisting of k-planes contained in X.

Theorem 1.1 will be proved in the next section and followed by the discussion of some of its implications. As we see in the course of the proof, we obtain the results of [HMP] on Fano varieties via a different approach.

I would like to thank J. Harris for suggesting this problem and proofreading the paper. Moreover, I have benefited greatly from discussions with him.

It has come to my attention that these results were also obtained independently by Debarre and Manivel.

2. Unirationality of Fano varieties. With the setup in Theorem 1.1, we have X a hypersurface of degree d in $\mathbb{P}^n$, and $F_k(X)$ its Fano variety. Let $M(k+1,n+1)$ be the linear space of $(k+1)\times(n+1)$ matrices, and $M_k(k+1,n+1) \subset M(k+1,n+1)$ be the determinantal variety of $(k+1)\times(n+1)$ matrices of rank $\leqslant k$. There is a natural smooth surjective morphism

$$\varphi\colon \mathbb{P}M(k+1,n+1)\backslash M_k(k+1,n+1) \to \mathbb{G}(k,n).$$

Take $W = \varphi^{-1}(F_k(X))$. It is not hard to see the following proposition.

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