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QUANTUM SUPERGROUPS OF $GL(n|m)$ TYPE: DIFFERENTIAL FORMS, KOSZUL COMPLEXES, AND BEREZINIANS

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0. Introduction

0.1. Short description of the paper

0.1.1. In this paper we will be concerned with differential Hopf algebras generated by sets of matrix elements. We start (Section 1) by giving a construction of such algebras, generalising the construction [31], [27] of a bialgebra generated by a single set of matrix elements (without differential) as a universally coacting bialgebra preserving several algebras generated by a set of coordinates. In our generalisation the data are morphisms in the category of graded differential complexes.

0.1.2. Given a Hecke R-matrix for a vector space V, we construct in this paper another Hecke R-matrix $\mathscr{R}$ for the space $W = V \oplus V$ equipped with the differential $d = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and the grading $\sigma\colon W \to W$, $\sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The matrix $\mathscr{R}$ is distinguished by the property

$$\mathscr{R}(d \otimes 1 + \sigma \otimes d) = (d \otimes 1 + \sigma \otimes d)\mathscr{R}.$$

0.1.3. The algebra H of functions on the quantum supergroup constructed from $\mathscr{R}$ is a $\mathbb{Z}$-graded differential coquasitriangular Hopf algebra (Section 2). In brief, it defines a differential quantum supergroup. A quotient of H is the $\mathbb{Z}_{\geqslant 0}$-graded differential Hopf algebra Ω of differential forms defined via R in [25, 26, 30, 35].

The classical version ($q = 1$) of this construction is: take a vector space V, add to it another copy of it with the opposite parity and consider the general linear supergroup of the $\mathbb{Z}/2$-graded space so obtained.

0.1.4. We introduce Koszul complexes for Hecke R-matrices in Section 3. They are $\mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{\geqslant 0}$-graded algebras with two differentials, D of degree $(1, 1)$ and D' of degree $(-1, -1)$. We calculate their anticommutator, called the Laplacian. The cohomology space of D is called the Berezinian. It generalizes the determinant, coinciding in the even case with the highest exterior power of V. The behaviour of Koszul complexes and Berezinians with respect to the Hecke sum

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