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EMBEDDED GENUS-2 SURFACES IN FOUR-MANIFOLDS

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1. Introduction. In this paper, we will study the effect of the following smooth operation on Donaldson invariants, and then apply the corresponding results to compute Donaldson series for certain families of simply connected, smooth, closed four-manifolds.

Let Z_1, Z_2 be smooth, closed, oriented four-manifolds with $b_2^+(Z_i) > 1$ and $b_2^+(Z_i) - b_1(Z_i)$ odd. Suppose that Z_1, Z_2 contain smoothly embedded genus-2 surfaces $\Sigma_1 \hookrightarrow Z_1$, $\Sigma_2 \hookrightarrow Z_2$ with $(\Sigma_1)^2 = -1$, and $(\Sigma_2)^2 = +1$. Let $X_i = Z_i \backslash nd(\Sigma_i)$. Let us fix a $\phi_0 \colon \Sigma_1 \to \Sigma_2$ diffeomorphism. Take a $\phi \colon \partial X_1 \to \partial X_2$ orientation-reversing diffeomorphism covering ϕ_0, and define $Z = X_1 \cup_\phi X_2$. Under some technical restrictions, we will prove a product formula (see Theorem 2.5 which expresses certain Donaldson invariants of Z in terms of Donaldson invariants of Z_1 and Z_2). Note that the diffeomorphism type of Z depends also on ϕ. In Theorem 4.6 we prove a generalization of Theorem 2.5. For each *admissible* diffeomorphism $\phi_0 \colon \Sigma_1 \to \Sigma_2$ (see Definition 4.1 for the definition of admissible), we compute *all* Donaldson invariants of Z in terms of ϕ_0 and the Donaldson invariants of Z_1 and Z_2.

We demonstrate the usefulness of the generalized product formula by proving a conjecture of Fintushel and Stern on Donaldson invariants of Horikawa surfaces [19], [5]. As a corollary, we prove the following.

THEOREM 1.1. *If two Horikawa surfaces are homeomorphic, then there is a homeomorphism which identifies their Donaldson invariants.*

As another application, we construct several families of symplectic manifolds, and compute their Donaldson invariants by using Theorem 4.6 and its close cousins Theorems 4.7 and 4.8. As a by-product, we produce hordes of simply connected, homeomorphic manifolds with identical Donaldson invariants. While this paper mainly deals with Donaldson theory, we also make a digression here and show that these four-manifolds have identical Seiberg-Witten invariants as well.

The paper is built in the following way. In Section 2, we review the simple type condition of Kronheimer and Mrowka, and state the main technical results. Section 3 contains the proof of Theorem 2.5. In Section 4, we study admissible diffeomorphisms and prove Theorem 4.6. The conjecture of Fintushel and Stern is proved in Section 5. The construction of the symplectic examples, and the computation of their Donaldson invariant, is given in Section 6. We finish the

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