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KAZHDAN-PATTERSON LIFTING FOR $GL(n,\mathbb{R})$

JEFFREY ADAMS AND JING-SONG HUANG

§1. Introduction. Kazhdan and Patterson have conjectured the existence of a lifting theory, taking representations of $GL(n,\mathbb{F})$ to representations of the nonlinear covering groups of $GL(n,\mathbb{F})$. We consider the case $\mathbb{F}=\mathbb{R}$. We work directly on the level of global characters.

Let $G=GL(n,\mathbb{R})$ and $p\colon \tilde{G}\to G$ be its nonlinear twofold cover (see §2 for details). For $s\colon G\to\tilde{G}$ a section, $g\to s(g)^2$ defines a map from conjugacy classes in G to conjugacy classes in $\tilde{G}$, independent of the choice of s. As in [6, §2], we modify this by a certain function $u: G\to\ker(p)$ to obtain $t(g)=s(g)^2u(g)$.

Let π be a virtual module for G with global character θ_π viewed as a function on the regular semisimple elements. For g, a regular semisimple element in the image of t, let

$$t_*(\theta_\pi)(g)=2^{[n/2]-n}\sum_{\{h\in G|t(h)=g\}}\frac{\Delta(h)}{\Delta(g)}\Theta_\pi(h), \tag{1.1}$$

where Δ is the usual Weyl denominator. Let

$$Z_0=\begin{cases} I & n \text{ even},\\ \pm I & n \text{ odd},\end{cases}\qquad \tilde{Z}_0=p^{-1}(Z_0); \tag{1.2}$$

this is a central subgroup of $\tilde{G}$. Fix a genuine character χ_0 of $\tilde{Z}_0$; there are two such choices if n is odd, and one if n is even. For $g\in t(G), z\in\tilde{Z}_0$ define

$$t_*(\theta_\pi)(gz)=t_*(\Theta_\pi)(g)\chi_0(z). \tag{1.3}$$

Finally, set $t_*(\Theta_\pi)(g)=0$ if g is not in the set $t(G)\tilde{Z}_0$.

The conjecture of Kazhdan and Patterson [6, end of §4] says that if π is irreducible, then $t_*(\pi)$ is an irreducible representation, up to sign, or is zero. Our main result is the following.

Theorem 1.4. *Let π be an irreducible unitary representation of G. Then $t_*(\pi)$ is either zero or an irreducible unitary representation, up to sign. It may be computed explicitly (Proposition 5.4).*

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