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QUASIRIGIDITY OF HYPERBOLIC 3-MANIFOLDS AND SCATTERING THEORY

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1. Statement of results. Geometrically finite Kleinian groups uniformizing infinite volume-hyperbolic 3-manifolds exhibit a rich deformation theory due to work of Ahlfors, Bers, Kra, Marden, Maskit, Thurston, and others. The purpose of this note is to introduce scattering theory as an analytic tool in the study of the deformations of complete geometrically finite hyperbolic structures. The results in this paper are restricted to a subclass of geometrically finite Kleinian groups called *convex cocompact groups*, that is, those containing no parabolic subgroups.

Assume that Γ is a convex cocompact, torsion-free Kleinian group with nonempty regular set $\Omega(\Gamma)$ (see Section 2 for definitions). The compact (possibly disconnected) quotient surface $\Omega(\Gamma)/\Gamma$ is the conformal boundary at infinity of the hyperbolic 3-manifold $M(\Gamma) = \mathbf{H}^3/\Gamma$. To the Laplacian Δ on $M(\Gamma)$, we can associate a scattering operator acting on sections of certain complex line bundles over the conformal boundary $\Omega(\Gamma)/\Gamma$. These sections are most conveniently described as automorphic forms on $\Omega(\Gamma)$. For a complex parameter s, let $\mathscr{F}_s(\Gamma)$ be the space of automorphic forms of weight s on $\Omega(\Gamma)$ (see Section 3 for the definition). The scattering operator $S(s)$ is a pseudodifferential operator [10] with known singularity mapping $\mathscr{F}_{2-s}(\Gamma) \to \mathscr{F}_s(\Gamma)$. For Re $s = 1$, $\mathscr{F}_s(\Gamma)$ possesses a natural L^2 inner product, so we can complete these spaces to form Hilbert spaces $\mathscr{H}_\sigma(\Gamma)$, where $s = 1 + i\sigma$.

Now take two convex cocompact, torsion-free Kleinian groups Γ_1, Γ_2 with $\Omega(\Gamma_i) \neq \varnothing$, $i = 1, 2$. Assume there exists an orientation-preserving diffeomorphism $\psi\colon \Omega(\Gamma_1) \to \Omega(\Gamma_2)$ that induces an isomorphism $\phi\colon \Gamma_1 \to \Gamma_2$. Let $S_i(s)$ be the scattering operator mapping $\mathscr{F}_{2-s}(\Gamma_i) \to \mathscr{F}_s(\Gamma_i)$ for $i = 1, 2$. We can pull $S_2(s)$ back, via the diffeomorphism ψ, to an operator $\psi^* S_2(s)\colon \mathscr{F}_{2-s}(\Gamma_1) \to \mathscr{F}_s(\Gamma_1)$. Perry [17] shows that if for some Re $s = 1, s \neq 1$, the operator

$$S_{\text{rel}}(s) = S_1(s) - \psi^* S_2(s)$$

is trace class as a map $\mathscr{H}_{-\sigma}(\Gamma_1) \to \mathscr{H}_\sigma(\Gamma_1)$, then $S_{\text{rel}}(s) = 0$ and the diffeomorphism ψ is actually a Möbius transformation; that is, the manifolds $M(\Gamma_i) = \mathbf{H}^3/\Gamma_i$ are isometric.

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