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MORDELL-WEIL GROUPS IN PROCYCLIC EXTENSIONS OF A FUNCTION FIELD

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Let K be a number field, p a prime number, and L the cyclotomic $\mathbf{Z}_p$ extension of K. Mazur made the following conjecture.

Conjecture [M]. *Given an elliptic curve E/K with good reduction at p, then the group of rational points $E(L)$ is finitely generated.*

In this paper, we consider a function field analogue of Mazur's conjecture. We show, in some special cases, that the Mordell-Weil group is finitely generated in certain towers of cyclic extensions of a 1-dimensional function field over $\mathbf{C}$. We prove the following.

Theorem 1. *Let $K = K_1 = \mathbf{C}(T)$, $K_r = \mathbf{C}(T^{1/r})$, and $L = \bigcup_{r=1}^{\infty} K_r$. Let E/K be an elliptic curve, and assume that away from $T = 0, \infty$, the reduction of E is either good or of multiplicative type. Further, we assume one of the following.*

(1) *Letting $n_P = |\mathrm{ord}_P(j_E)|$, then*

$$\sum_{P \neq 0, \infty, n_P \neq 0} (1 - n_P/6) < 1;$$

or

(2) *E has multiplicative reduction at $T = 1, -1$, there are no other primes of bad reduction, and the automorphism τ of $\mathbf{C}(T)$ defined by $\tau(T) = 1/T$ lifts to an automorphism of E.*

Then $E(L)$ is finitely generated.

Remark. Such curves arise in natural contexts. For example, the elliptic curve over $\mathbf{C}[T]$ defined by the Weierstrass equation

$$Y^2 = 4X^3 - 27XT - 27T$$

is (up to an isomorphism of the base) the elliptic modular surface associated to $PSL(2, \mathbf{Z})$ (see [Sh1]) and satisfies condition (1). The curve

$$Y^2 = 4X^3 - 3X + (T + 1/T)$$

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