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GEOMETRIC CONSTRUCTION OF CRYSTAL BASES

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1. Introduction

1.1. G. Lusztig [L3] gave a realization of the quantized universal enveloping algebras as the Grothendieck group of a category of perverse sheaves of the quiver variety. Let (I, Ω) be a finite oriented graph (= quiver), where I is the set of vertices and Ω is the set of arrows. Let us associate a complex vector space V_i to each vertex $i \in I$. We set

$$E_{V,\Omega} = \bigoplus_{\tau \in \Omega} \operatorname{Hom}(V_{\mathrm{out}(\tau)}, V_{\mathrm{in}(\tau)})$$

and

$$X_V = E_{V,\Omega} \oplus E^*_{V,\Omega}.$$

They are finite-dimensional vector spaces with the action of the algebraic group $G_V = \prod_{i \in I} GL(V_i)$. We regard X_V as the cotangent bundle of $E_{V,\Omega}$. Lusztig [L3] realized a half of the quantized universal enveloping algebra $U_q^-(\mathfrak{g})$ as the Grothendieck group of $\mathscr{Q}_{V,\Omega}$. Here $\mathscr{Q}_{V,\Omega}$ is a subcategory of the derived category $D_c^b(E_{V,\Omega})$ of the bounded complexes of constructible sheaves on $E_{V,\Omega}$. The irreducible perverse sheaves in $\mathscr{Q}_{V,\Omega}$ form a base of $U_q^-(\mathfrak{g})$, which is called canonical basis.

In [L5] he stated the following problem.

Problem 1. If the underlying graph is of type A, D, or E, then the singular support of any canonical base is irreducible.

One of the purposes of this paper is to construct a counterexample of this problem for type A.

1.2. Let G be a connected complex semisimple algebraic group, B a Borel subgroup of G, and $X = G/B$ the flag variety. Let D_X denote the sheaf of differential operators on X. We denote the half sum of positive roots by ρ and the Weyl group by W. For $w \in W$, let M_w be the Verma module with highest weight $-w(\rho) - \rho$ and L_w its simple quotient. By the Beilinson-Bernstein correspondence, M_w and L_w correspond to regular holonomic D_X-modules $\mathfrak{M}_w$ and $\mathfrak{L}_w$ on X, respectively. The characteristic varieties $\mathrm{Ch}(\mathfrak{M}_w)$ and $\mathrm{Ch}(\mathfrak{L}_w)$ are Lagrangian

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