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ON A GENERALIZATION OF THE DOUBLE COSET FORMULA

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CONTENTS

The double coset formula is a very useful tool in computations of group homology for finite groups. In this paper we prove a generalization to the case of infinite index subgroups, using locally finite homology. In this context a transfer can be constructed (see [5]), and a double coset formula, which is very similar to the usual one, holds.

As an application, we prove an estimate for the rank of the homology at the *vcd* of a congruence subgroup of $Sl_n(\mathbb{Z})$. More precisely, let $\Gamma(n,p)$ be the subgroup of $Sl_n(\mathbb{Z})$ consisting of matrices congruent to identity mod p, $n \geqslant 2$, $p \geqslant 3$, p a prime. Then it is known (see [4]) that the group $H_{(n(n-1))/2}(\Gamma(n,p),\mathbb{Z})$ is a finitely generated free abelian group. The result is the following theorem.

THEOREM 1. *The rank of* $H_{(n(n-1))/2}(\Gamma(n,p),\mathbb{Z})$ *is at least* $\geqslant p^{\binom{n}{2}}((p-1)/2)^{n-1}$.

This improves the result in [6] by the factor $((p-1)/2)^{n-1}$. While the above estimate can be derived without using our double coset formula, we believe this setting makes the main idea more apparent.

Our estimate is closely related to the results in the following papers: [3], where the linear independence of the homology classes we construct is proven; [1], where it is shown that the rank in question is, in principle, computable for $n \leqslant 4$; and [2], where an upper bound is given for all ns.

1. Locally finite homology and infinite transfers. For a space X, denote by $C_k^{lf}(X)$ the set of formal sums, perhaps infinite, of k-simplices with integer coefficients such that any compact set $K \subseteq X$ intersects finitely many simplices which appear with a nonzero coefficient. The locally finite homology of X, which we denote by $H_*^{lf}(X)$, is defined to be the homology of the chain $C_*^{lf}(X)$. As an example, $H_k^{lf}(\mathbb{R}^n)$ is zero if $k \neq n$ and $\mathbb{Z}$ if $k = n$. Also, for X compact we have $H_*^{lf}(X) \simeq H_*(X)$.

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