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A TOWER OF THETA CORRESPONDENCES FOR G_2

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Introduction. Let G be a simple, simply laced, split algebraic group, defined over a number field F. Let L and H be two reductive groups, defined over F, such that $L \times H$ embeds in G (over F). We say that (L, H) is a commuting pair inside G. (If $L = C_G(H)$ and $H = C_G(L)$, then (L, H) is said to be a reductive dual pair inside G. The classification of such pairs was started by Dynkin [D] and recently completed by Rubenthaler [R].) In this work, we consider the theta correspondence associated to certain pairs (L, H) inside G, and we obtain new examples of (functorial) lifts. Here we use the automorphic realization θ_G of the theta (minimal) representation, as constructed and studied in our earlier work [GRS1]. The minimal representation was constructed over a local non-archimedean field by Kazhdan and Savin [K], [KS], [S] for the various cases mentioned here. Thus, let π be an irreducible, automorphic, cuspidal representation of $L_{\mathbb{A}}$ (with $\mathbb{A}$ being the ring of adèles of F) given with a specific realization in the space of cusp forms on $L(F)\backslash L(\mathbb{A})$. Let f_{θ_G} be an automorphic form in the space of θ_G. The theta lift of π is the automorphic representation of $H_{\mathbb{A}}$, $\theta_G^H(\pi)$, whose space is spanned by

$$h \mapsto \int_{L_F\backslash L_{\mathbb{A}}} f_{\theta_G}(g,h)\varphi(g)\,dg$$

as φ varies in V_π and f_{θ_G} varies in θ_G. Since we consider an automorphic representation together with a concrete realization, we sometimes use the same notation for the representation and its space. Alternatively, if τ is an automorphic representation, we sometimes denote its space by V_τ. We omit H from $\theta_G^H(\pi)$ when H is understood. When G is replaced by the double cover of the symplectic group, and θ_G is replaced by the classical Weil representation, realized by theta series, we get the well-known theta correspondence and the related local theory of Howe correspondence, which has proved, so far, to be very fruitful in establishing many examples of functorial liftings which appear together with related periods and related L-functions. The pairs that we study here fit into a tower similar to the one obtained by Rallis [Ra] for pairs of type (Sp_n, O_ℓ). The tower is as follows. The first member of the pair (L, H) is $L = G_2$, and H belongs to

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