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COMPACT EINSTEIN-WEYL MANIFOLDS WITH LARGE SYMMETRY GROUP

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1. Introduction. The Einstein-Weyl equations are a conformally invariant generalisation of the Einstein equations, introduced by Weyl [27]. They have been thoroughly studied in dimension 3 [4], [8], [10], [12], [24], [25], [26], where it is known that any solution on a compact manifold is either a compact quotient of hyperbolic 3-space $\mathscr{H}^3$ or has a cohomogeneity-one action of the 2-torus T^2. Furthermore, in any dimension, a compact Einstein-Weyl manifold that is not Einstein has a nontrivial symmetry [25]. To find new examples in higher dimensions, it is therefore natural to look for solutions with a high degree of symmetry.

In this paper we will give a full classification of the compact 4-dimensional Einstein-Weyl structures for which the symmetry group is at least 4-dimensional. Restricting to dimension 4 allows us to take advantage of various topological consequences of the Einstein-Weyl equations [23], [21], [7]. The assumption that the group of symmetries is at least 4-dimensional implies that the solutions are either homogeneous or have cohomogeneity one. Our results also sharpen previous results [9], [2] on 4-dimensional Einstein metrics (Theorem 3.1) and correct the topological classification [20] of cohomogeneity-one 4-manifolds (Remark 6.4).

Let $(M, [g])$ be a conformal manifold. A torsion-free connection D preserving the conformal class $[g]$ is called a *Weyl connection.* Fixing a choice of Riemannian metric g in the conformal class, we obtain a 1-form ω from the equation $Dg = \omega \otimes g$. Conversely, the 1-form ω, together with the Levi-Civita connection ∇ of g, determines D by

$$D = \nabla - \frac{1}{2}(\omega \veebar \mathrm{Id} - g \otimes \omega^\sharp),$$

where $\omega^\sharp$ is the vector field such that $\omega = g(\omega^\sharp, \cdot)$, and $(\omega \veebar \mathrm{Id})(X, Y) = \omega(X)Y + \omega(Y)X$. Under a conformal change $g \mapsto \exp(\lambda)g$, we have $\omega \mapsto \omega + d\lambda$, and so it makes sense to call D *closed* if $d\omega = 0$ and *exact* if ω is exact.

The Einstein-Weyl equations state

$$Sr^D = \Lambda g,$$

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