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BOUNDED COHOMOLOGY AND TOPOLOGICALLY TAME KLEINIAN GROUPS

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The notion of bounded cohomology was introduced by Gromov [5]. The bounded cohomology $H_b^*(X;\mathbf{R})$ for any topological space X is defined with the subcomplex $C_b^*(X)$ of the usual cochain complex $C^*(X)$ consisting of bounded cochains. This cohomology has the advantage of admitting the naturally defined pseudonorm $\|\cdot\|$. According to Soma [18], in general, this pseudonorm is not a norm. So, we also consider the quotient space

$$HB^*(X;\mathbf{R}) = H_b^*(X;\mathbf{R})/\{[c] \in H_b^*(X;\mathbf{R}); \|[c]\| = 0\},$$

and denote the element of $HB^*(X;\mathbf{R})$ corresponding to $[c] \in H_b^*(X;\mathbf{R})$ by $[c]_B$. Note that $HB^*(X;\mathbf{R})$ is a Banach space with the norm $\|\cdot\|$.

The second bounded cohomology of a closed surface of genus $g > 1$ was studied by Brooks-Series [2], Mitsumatsu [11], and Barge-Ghys [1], and the third by Yoshida [22] and Soma [17]. Here, we will study further the third bounded cohomology and connections with hyperbolic 3-manifolds.

For a torsion-free Kleinian group Γ, the bounded 3-cocycle ω_Γ on the hyperbolic 3-manifold $M_\Gamma = \mathbf{H}^3/\Gamma$ is defined by $\omega_\Gamma(\sigma) = \Omega_\Gamma(\text{straight}(\sigma))$ for any singular simplex $\sigma\colon \Delta^3 \longrightarrow M_\Gamma$, where Ω_Γ is the volume form on M_Γ. In fact, since the volume of any straight simplex is less than the volume $\mathbf{v}_3$ of a regular ideal simplex v_0 in $\mathbf{H}^3$, and since v_0 is well approximated by a usual straight simplex in $\mathbf{H}^3$, the norm $\|\omega_\Gamma\|$ is equal to $\mathbf{v}_3$. So, the *fundamental class* $[\omega_\Gamma] \in H_b^3(M_\Gamma;\mathbf{R})$ of M_Γ has the pseudonorm $\|[\omega_\Gamma]\| \leqslant \mathbf{v}_3$. In [17], we were mainly concerned with Kleinian groups, Γ, isomorphic to closed surface groups and such that the injectivity radii $\text{inj}(M_\Gamma) = \{\text{inj}_{M_\Gamma}(x); x \in M_\Gamma\} > 0$, in which Minsky's ending lamination theorem [10] played an important role. In particular, this theorem was used effectively to prove rigidity theorems [20, Theorems A and D] for certain hyperbolic 3-manifolds M_Γ in terms of the "distance" between the fundamental classes $[\omega_\Gamma]$ with respect to the pseudonorm.

In this paper, we consider the case where Γ are topologically tame Kleinian groups. Our proofs here are based on Canary's results about topologically tame Kleinian groups in [3] and his covering theorem in [4].

THEOREM 1. *Suppose that Γ is a topologically tame Kleinian group such that the volume of M_Γ is infinite. Then $[\omega_\Gamma] = 0$ in $H_b^3(M_\Gamma;\mathbf{R})$ if and only if Γ is either*

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