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v_n-ELEMENTS IN RING SPECTRA AND APPLICATIONS TO BORDISM THEORY

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Introduction. The work of Hopkins and Smith [HS] has shown that the stable homotopy category has layered periodic behavior. On the (p-local) sphere, the only non-nilpotent self-maps are multiplication by a power of p. But if we kill such a power to form the Moore space $M(p^k)$, then we get a new family of non-nilpotent self-maps, called the v_1 self-maps. Similarly, if we kill one of those, we get v_2 self-maps, and this behavior continues.

One of the great advantages of the Brown-Peterson spectrum BP is that the periodicities are not layered, but they all appear as homotopy classes $v_n \in \pi_{2(p^n-1)}BP$. Another great advantage of BP is that it is comparatively simple algebraically. Its coefficient ring is polynomial, and it is possible to calculate in the Adams-Novikov spectral sequence based on the operations in BP-homology. In fact, most of the spectra used by algebraic topologists are complex-oriented, in that they admit maps of ring spectra from BP. But there is one crucial example that does not, namely, real K-theory KO. Hopkins and Miller [HMi] have recently shown that KO is the tip of an iceberg of noncomplex-oriented theories that have interesting torsion.

It would be nice to have a bordism spectrum that did admit maps to the Hopkins-Miller theories EO_n. Recall that $MO\langle k\rangle$ is the Thom spectrum arising from the $k-1$-connected Postnikov cover $BO\langle k\rangle$ of BO. Similarly, $MU\langle k\rangle$ is the Thom spectrum arising from the $k-1$-connected Postnikov cover of BU. Note that $MO\langle 4\rangle = MSpin$ and $MU\langle 4\rangle = MSU$ both admit orientations to KO. We hope that this is also the beginning of a general phenomenon, and that the $MO\langle k\rangle$ and $MU\langle k\rangle$ will admit orientations to EO_n when k is sufficiently large.

Such an orientation may have some analytic meaning. Witten interprets a (conjectural) orientation from $MO\langle 8\rangle$ to elliptic cohomology, which should be EO_2, as the index of an S^1-equivariant Dirac operator on the free loop space of a manifold with $MO\langle 8\rangle$-structure. However, it will be hard to get at the algebraic meaning of such an orientation because we know so little about the $MO\langle k\rangle$.

This paper is an attempt to get some qualitative understanding of the $MO\langle k\rangle$ and $MU\langle k\rangle$. We try to find analogues of v_n in the homotopy of $MO\langle k\rangle$ and $MU\langle k\rangle$. It turns out to be easier to study the existence and properties of such analogues in general ring spectra. This is the content of the first section. This section owes much to the beautiful paper [HS], and discusses many of the same ideas. The highlight of this section is a sufficient condition for the existence of

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