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DISSOLVING A CUSP FORM IN THE PRESENCE OF MULTIPLICITIES

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1. Introduction. Let Γ be a discrete, cofinite, but not cocompact, subgroup of the orientation-preserving isometries of the hyperbolic plane $\mathbf{H}^2$. The hyperbolic Laplacian $\Delta = -y^2(\partial_x^2 + \partial_y^2)$ acting on Γ-automorphic functions has a continuous spectrum on $[1/4, \infty)$ of multiplicity equal to the number of singular cusps, a discrete "exceptional" spectrum in $[0, 1/4)$, and possibly a nonempty pure point spectrum embedded in the continuous spectrum. The eigenfunctions associated with the embedded pure point spectrum are known in the theory of automorphic forms as *Maass cusp (wave) forms* [V].

Let $N(x)$ denote the number of Maass cusp forms with eigenvalues less than x. Let $M(x)$ denote the number of poles of the determinant of the *scattering matrix* (see [LP]) with modulus less than $\sqrt{x}$. Selberg proved the following generalization of Weyl's law:

$$N(x) + M(x) \sim \frac{\text{Area}(\Gamma\backslash\mathbf{H}^2)}{4\pi} \cdot x \tag{1}$$

as x tends to infinity. Selberg also proved that for *congruence subgroups* of $SL(2, \mathbf{Z})$, one has $(M(x))/x \sim 0$ [Se]. A group Γ with this latter property will be called *essentially cuspidal* (see [Sa] and [PS2]).

For a long time it was supposed that all discrete subgroups were essentially cuspidal. As yet this conjecture has not been disproven, but several recent results indicate that the conjecture is unlikely to be true: see [C], [PS1], [DIPS], [W2], [J1], [PS2], and [P]. In all of these papers, it is shown that enough Maass cusp form eigenvalues *dissolve* into the continuous spectrum, so that the asymptotic contribution of $M(x)$ increases. This means that *resonances* are created in the plane $Re(s) < 1/2$, where $\lambda = s(1-s)$ is the usual spectral parameter.

The most recent evidence, [W2], [PS2], [J2], and [P], is based on various applications of the strategy developed in [PS1]. This strategy revolves around the integral

$$\int_{\Gamma\backslash\mathbf{H}^2} E \cdot \dot{\Delta}\phi \, \frac{dx\,dy}{y^2}. \tag{2}$$

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