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ON FUJITA'S CONJECTURE

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1. Introduction. Let X be a smooth projective variety over an algebraically closed field $\Bbbk$ of characteristic 0. The aim of this paper is to find conditions for an ample divisor L such that there is a section of $\mathcal{O}_X(K_X + L)$ which does not vanish at a given point $x \in X$, where K_X denotes the canonical divisor. Let us illustrate this in two examples.

The first example is the case $\dim X = 1$. Then the ideal sheaf $\mathcal{I}_x$ of x is locally free of degree -1. Therefore, by Serre duality, the first cohomology of $\mathcal{I}_x(K_X + L)$ vanishes if $\deg L > 1$, and from a long exact cohomology sequence, we find that $\mathcal{O}_X(K_X + L)$ is globally generated. By the same argument, we see that if $M = \{x\}$, so that $\deg M = 1$, then x is a base point of $\mathcal{O}_X(K_X + M)$, which just means that there are no differential forms with one first-order pole because of the residue formula. In this case $\deg(K_X + M) = 2g - 1$, where g is the genus of X. This example shows that there are divisors on curves of arbitrary high degree which are not globally generated. This is the reason for considering global generation of adjoint bundles $\mathcal{O}_X(K_X + L)$ rather than that of $\mathcal{O}_X(L)$ itself.

The second standard example is the projective space $X = \mathbf{P}^n(\Bbbk)$. Then every divisor on X is linear equivalent to mH, where H is a hyperplane section. In particular, $K_X \equiv -(n+1)H$. Therefore $\mathcal{O}_X(K_X + mH)$ is globally generated if and only if $m > n$. Generalizing these two examples, T. Fujita raised in [7] the following conjecture.

CONJECTURE 1.1 (Fujita). *Let X be a smooth projective variety of dimension n, and let H be an ample divisor on X. Then $\mathcal{O}_X(K_X + mH)$ is generated by its global sections if $m > n$ or if $m = n$ and $H^n > 1$.*

As we have seen above, this is true for curves. But our argument depends

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