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MODULI OF EQUIVARIANT ALGEBRAIC VECTOR BUNDLES OVER A PRODUCT OF AFFINE VARIETIES

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§0. Introduction. In this paper, we consider the base field $\mathbb{C}$ of complex numbers. Let G be a reductive affine algebraic group and X a G-stable affine cone in a G-module. We denote by $Vec_G(X, Q)$ the set of algebraic G-vector bundles over X whose fiber at the origin (the summit of the cone) is a G-module Q, and by $VEC_G(X, Q)$ the set of G-isomorphism classes in $Vec_G(X, Q)$. We denote by $[E]$ the isomorphism class of $E \in Vec_G(X, Q)$. The equivariant Serre problem asks whether $VEC_G(X, Q)$ is trivial when X is a G-module. When G is abelian and X is a G-module, Masuda, Moser-Jauslin, and Petrie [MMP1] have shown that $VEC_G(X, Q)$ is trivial. However, for a nonabelian group G, $VEC_G(X, Q)$ is not trivial even if X is a G-module (see [S], [Kn], [MP], [MMP2]). Little is known on the moduli space $VEC_G(X, Q)$, especially when the dimension of the algebraic quotient space $X//G$ is greater than one. Even if X is a G-module, to classify elements in $VEC_G(X, Q)$, when $\dim X//G \geqslant 2$, is an open problem. When X is a G-module with one-dimensional quotient, Schwarz [S] showed that $VEC_G(X, Q) \cong \mathbb{C}^p$ for a nonnegative integer p (for details, see Kraft and Schwarz [KS]). The result of Schwarz [S] can be extended to the case where X is a G-stable affine cone with smooth one-dimensional quotient in a G-module. More generally, when X is a weighted G-cone with smooth one-dimensional quotient [MMP3] (see §1 for the definition), we have the following theorem.

Theorem A ([M1]; cf. [S], [KS]). *Let X be a weighted G-cone with smooth one-dimensional quotient and Q be a G-module. Then $VEC_G(X, Q) \cong \mathbb{C}^p$ for a nonnegative integer p. Moreover, there is a G-vector bundle $\mathfrak{B}$ over $X \times \mathbb{C}^p$ with fiber Q such that the map $\mathbb{C}^p \ni z \mapsto [\mathfrak{B}|_{X \times \{z\}}] \in VEC_G(X, Q)$ gives a bijection.*

Let X, p, and $\mathfrak{B}$ be as in Theorem A. We denote by $\mathrm{Mor}(\mathbb{A}^m, \mathbb{C}^p)$ the set of morphisms from affine m-space $\mathbb{A}^m$ to $\mathbb{C}^p$. Then there is a map

$$\Phi\colon \mathrm{Mor}(\mathbb{A}^m, \mathbb{C}^p) \to VEC_G(X \times \mathbb{A}^m, Q)$$

defined by $\Phi(f) = [(\mathrm{id}_X \times f)^*\mathfrak{B}]$ for $f \in \mathrm{Mor}(\mathbb{A}^m, \mathbb{C}^p)$. By Theorem A, it is bijective when $m = 0$. Moreover, Theorem A implies that Φ is injective. Masuda and Petrie have shown that Φ is bijective in some examples. In [M2], we showed that Φ is bijective when Q is multiplicity free with respect to a principal isotropy

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