Vol. 88, No. 1 DUKE MATHEMATICAL JOURNAL © 1997

UNIFORM BOUNDEDNESS FOR RATIONAL POINTS

PATRICIA L. PACELLI

1. Introduction. A well-known conjecture made by S. Lang states that if K is a number field, then the set of K-rational points of any variety of general type defined over K is not dense in the Zariski topology. In a recent paper entitled "Uniformity of Rational Points" [CHM], L. Caporaso, J. Harris, and B. Mazur show that this conjecture implies the existence of a uniform bound on the number of K-rational points over all smooth curves of genus g defined over K, for fixed $g \geqslant 2$. Their bound depends on the genus g and on the number field K.

D. Abramovich has proved an extension of this result. In his paper [A] he proves that, assuming Lang's conjecture, the bound $B(K, g)$ of [CHM] remains bounded as K varies over all quadratic extensions of a fixed number field.

It is this result of Abramovich's that we shall generalize in this paper. We will prove that given a number field K, Lang's conjecture implies the existence of a uniform bound on the number of L-rational points over all smooth curves of a fixed genus $g > 1$ defined over L, as L varies over all extensions of K of degree d for any positive integer d. This bound will depend on K, d, and g, but is independent of the actual number field L.

THEOREM 1.1. *Assume that Lang's conjecture regarding varieties of general type is true. Let $g \geqslant 2$ and $d \geqslant 1$ be integers, and let K be a number field. Then there exists an integer $B_K(d, g)$ which, for a given K, depends only on d and g, such that for any extension L of K of degree d, and any curve C of genus g defined over L, it follows that*

$$\#C(L) \leqslant B_K(d, g).$$

By letting $K = \mathbf{Q}$ we have the following result.

COROLLARY 1.2. *Assume Lang's conjecture is true. Let $g \geqslant 2$ and $d \geqslant 1$ be integers. Then there exists a bound $B(d, g)$, depending only on d and g, such that for any number field L of degree d, and for any curve C of genus g defined over L, it follows that*

$$\#C(L) \leqslant B(d, g).$$

Since any extension L of K of degree d is a number field of some fixed degree d', Theorem 1.1 and Corollary 1.2 are equivalent.

Received 30 January 1996.