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ON LIVŠIC'S THEOREM, SUPERRIGIDITY, AND ANOSOV ACTIONS OF SEMISIMPLE LIE GROUPS

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1. Introduction. During the last decade, Anosov actions of semisimple Lie groups and their lattices have become a focal point in the study of rigidity properties of such groups. Most importantly, local smooth rigidity has been established for various standard algebraic Anosov actions by a number of authors [15], [11], [13], [12], [14], [19], [20], [21]. The conjecture arose that all such actions are essentially C^∞-conjugate to algebraic actions. The proof of this conjecture for the special class of Cartan actions is the goal of the current paper and its sequel [8].

In this paper, we introduce an additional geometric structure for certain types of volume-preserving Anosov actions of a connected higher-rank semisimple Lie group G of the noncompact type on a closed manifold M. More precisely, we will find a Hölder Riemannian metric, a Hölder splitting $\bigoplus E_\lambda$ of the tangent bundle, and a finite-dimensional representation π of G such that the elements in a Cartan subgroup of G expand and contract vectors in E_λ *precisely* according to a weight of π. Note that Zimmer's superrigidity theorem for cocycles yields the same conclusion with respect to a measurable Riemannian metric. The difference in regularity, however, is crucial to our classification of such actions.

One fundamental tool required to obtain these geometric results is a generalization of celebrated work of Livšic on the cohomology of Anosov systems. Livšic showed in particular that an $\mathbb{R}$-valued Hölder cocycle that is measurably cohomologous to the trivial cocycle is Hölder cohomologous to the trivial cocycle [16]. Livšic also obtained results for cocycles taking values in nonabelian Lie groups under the additional assumption that the cocycle evaluated on generators takes values sufficiently close to the identity. Theorem 2.1 and its corollaries provide bundle theoretic versions of Livšic's theorem, as we will now explain. Suppose a group G acts on a principal H-bundle $P \to M$ via bundle automorphisms. Since any bundle has a measurable section, such an action is measurably isomorphic to a skew product action on $M \times H$ via a cocycle $\alpha: G \times M \to H$. Different trivializations of P correspond to distinct yet cohomologous cocycles. Note that α is measurably cohomologous to the trivial cocycle precisely when there is a measurable G-invariant section of P. More

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