

CANONICAL BASES IN TENSOR PRODUCTS AND GRAPHICAL CALCULUS FOR $U_q(\mathfrak{sl}_2)$

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Introduction. Finite-dimensional representations of the Lie algebra \mathfrak{sl}_2 is one of the most studied subjects in representation theory. Numerous formulas and identities are known for the structure constants of tensor products of irreducible representations, namely, for Clebsch-Gordan ($3j$ -symbols) and Racah-Wigner

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