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CONVOLUTION BODIES AND THEIR LIMITING BEHAVIOR

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1. Introduction. Throughout this paper K and L denote convex symmetric bodies in $\mathbb{R}^n$. Our notation will be the standard notation that can be found, for example, in [7] or [9]. We start with the following definition.

Definition 1.1. For $0 < \delta < 1$, the *convolution body* of parameter δ of the convex symmetric bodies K and L is the set

$$C(\delta; K, L) = \{x \in \mathbb{R}^n \colon \mathrm{vol}_n(K \cap (x + L)) \geqslant \delta \,\mathrm{vol}_n(K \cap L)\}.$$

Also, if there exists a *normalization exponent* $\alpha > 0$ such that the limit

$$\lim_{\delta \to 1^-} \frac{C(\delta; K, L)}{(1-\delta)^\alpha}$$

is a nondegenerate set, we call this limit the *limiting convolution body* of K and L, and we denote it by $C(K, L)$.

Note that the boundaries $bd(C(\delta; K, L))$, $0 < \delta < 1$, are the level sets of the standard convolution of the characteristic functions $\chi_K(x)$ and $\chi_L(x)$ of the sets K and L.

We understand convergence as convergence (in Hausdorff sense) of the intersections of our sets with any (fixed) Euclidean ball in $\mathbb{R}^n$. By *degenerate set* we mean a set with an empty interior. The case of an infinite cylinder is considered to be a nondegenerate case. For every $0 \leqslant \alpha_0 \leqslant 1/2$ there are examples for which the limiting convolution body collapses to a point for all normalization exponents $\alpha < \alpha_0$, and it converges to an infinite cylinder (or $\mathbb{R}^n$), for all $\alpha \geqslant \alpha_0$ (see Example 3.14).

It is immediate that the δ-convolution body $C(\delta; K, L)$ as well as the limiting convolution body $C(K, L)$ of K and L are convex symmetric bodies (the convexity is a consequence of the Brunn-Minkowski inequality) and $C(\delta; K, L) = C(\delta; L, K)$, $C(K, L) = C(L, K)$. For $K = L$ the convolution bodies and their limiting behavior was studied by Schmuckenschläger in [8] where he proved that if

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