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CHOW GROUPS OF PROJECTIVE VARIETIES OF VERY SMALL DEGREE

HÉLÈNE ESNAULT, MARC LEVINE, AND ECKART VIEHWEG

Let k be a field. For a closed subset X of $\mathbb{P}^n_k$, defined by r equations of degree $d_1 \geqslant \cdots \geqslant d_r$, one has the numerical invariant

$$\kappa = \left[\frac{n - \sum_{i=2}^r d_i}{d_1}\right],$$

where $[\alpha]$ denotes the integral part of a rational number α. If k is the finite field $\mathbb{F}_q$, the number of k-rational points verifies the congruence

$$\#\mathbb{P}^n(\mathbb{F}_q) \equiv \#X(\mathbb{F}_q) \bmod q^\kappa,$$

while, if k is the field of complex numbers $\mathbb{C}$, one has the Hodge-type relation

$$F^\kappa H^i_c(\mathbb{P}^n_{\mathbb{C}} - X) = H^i_c(\mathbb{P}^n_{\mathbb{C}} - X) \quad \text{for all } i$$

(see [12], [5] and the references given there). These facts, together with various conjectures on the cohomology and Chow groups of algebraic varieties, suggest that the Chow groups of X might satisfy

$$\mathrm{CH}_l(X) \otimes \mathbb{Q} = \mathrm{CH}_l(\mathbb{P}^n_k) \otimes \mathbb{Q} = \mathbb{Q} \tag{$*$}$$

for $l \leqslant \kappa - 1$ (compare with Remark 5.6 and Corollary 5.7).

This is explicitly formulated by V. Srinivas and K. Paranjape in [16, Conjecture 1.8]; the chain of reasoning goes roughly as follows. Suppose X is smooth. One expects a good filtration

$$0 = F^{j+1} \subset F^j \subset \cdots \subset F^0 = \mathrm{CH}^j(X \times X) \otimes \mathbb{Q},$$

whose graded pieces F^l/F^{l+1} are controlled by $H^{2j-l}(X \times X)$ (see [10]). According to Grothendieck's generalized conjecture [8], the groups $H^i(X)$ should be generated by the image under the Gysin morphism of the homology of a codimension-κ subset, together with the classes coming from $\mathbb{P}^n$. Applying this to the diagonal in $X \times X$ should then force the triviality of the Chow groups in the desired range.

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