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THE TOTALLY GEODESIC RADON TRANSFORM ON THE LORENTZ SPACE OF CURVATURE −1

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1. Introduction. The Radon transform is heavily studied in a number of different settings nowadays. The spectrum of the investigations spread from the discrete case to the higher-rank symmetric spaces, but somewhat surprisingly, the pseudo-Riemannian spaces have not yet received much attention. The only works I know about are [3], [5].

We consider the isotropic Lorentz space $\mathscr{L}^n$ of signature $(1, n-1)$ with constant curvature -1, where n is the dimension of $\mathscr{L}^n$. This work depends basically on the observation that this space has a rotational $O(n)$ symmetry around its ideal points. (There are two ideal points, and every timelike geodesic reaches these points at $+\infty$ and $-\infty$.) The other crucial fact that we use is the geodesic correspondence between $\mathbb{R}^n$ and $\mathscr{L}^n$. Here, one has to note that a Lorentzian with geodesic correspondence is isotropic, hence harmonic, and therefore of constant curvature according to Lichnerowicz and Walker (see [5]).

In Section 2 we present our model for $\mathscr{L}^n$. We use the quadratic hypersurface model of Helgason [5] and project it onto a hyperplane orthogonal to the rotational axis. We determine explicitly the arc-length on the geodesics and the Haar measure of the isotropy group at all points.

Using the results of Section 2, the third section gives explicitly the Radon transform for spacelike and timelike total geodesics, respectively. The dual Radon transform is also calculated according to the "spacelike" and the "timelike" part of the Lorentz sphere at the given point. Finally, the spherical harmonic expansions of these transforms are shown.

In Section 4 we exhibit a kind of intertwining operators between the Euclidean and the Lorentzian Radon transform. Using this connection, we invert the Lorentzian Radon transform on a certain space of even functions and give a support theorem and range description. Then we prove that the spacelike Radon transform and the timelike Radon transform in even dimensions are also invertible.

The last section contains the necessary modification of the results of Section 4 for dimension two and considers the role of the odd functions that, contrary to the higher dimensions, are not annihilated in this case by the "timelike" Radon transform.

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