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FOURIER TRANSFORM AND THE IWAHORI-MATSUMOTO INVOLUTION

SAM EVENS AND IVAN MIRKOVIĆ

0. Introduction. The local Langlands program conjectures a classification of smooth, admissible representations of reductive p-adic groups by orbits of groups on certain geometric parameter spaces. This classification has been realized for unramified representations through work of Kazhdan and Lusztig [KL] (see also [Gi] or [CG]). This paper computes the action of the Iwahori-Matsumoto involution on unramified representations in terms of Fourier transform on the associated parameter space. The Iwahori-Matsumoto involution plays a role for p-adic groups analogous to the role played by the sign representation in the representation theory of Weyl groups. The relation between Fourier transform and the Iwahori-Matsumoto involution was suggested by Lusztig. For $GL(n)$, this result is a special case of a theorem of Moeglin and Waldspurger concerning the Zelevinsky involution.

The category of unramified representations of a split reductive group is equivalent to the category of representations of the Hecke algebra associated to the Iwahori subgroup, which is a specialization of a certain affine Hecke algebra. We will work with the affine Hecke algebra and its graded version in this paper. Let G be the reductive complex algebraic group, which is the Langlands dual of the split reductive group considered above. Let $\mathscr{H}$ be the associated affine Hecke algebra, an algebra over Laurent polynomials $\mathbb{C}[v, v^{-1}]$, and let $\mathfrak{g}$ be the Lie algebra of G. Irreducible representations of $\mathscr{H}$ in which v does not act by a root of unity are indexed by quadruples (s, v_0, x, ρ) up to G-conjugacy, where s is a semisimple element of G, $v_0 \in \mathbb{C}^*$ is not a root of unity, x is nilpotent in $\mathfrak{g}$ with $Ad(s)x = {v_0}^2 \cdot x$, and ρ is an irreducible representation of the component group of the centralizer of x appearing in the Springer module of x [KL] (see [Gr] for the root of unity case). We call the pair (s, v_0) an infinitesimal character of the corresponding representation. The centralizer $Z_G(s)$ acts with finitely many orbits on the vector space

$$\mathfrak{g}(s, v_0) = \{x \in \mathfrak{g} | Ad(s)x = v_0^2 \cdot x\},$$

and there is a bijection between irreducible representations of $\mathscr{H}$ with infinitesimal character (s, v_0) and $Z_G(s)$-equivariant irreducible perverse sheaves on $\mathfrak{g}(s, v_0)$

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