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C^2 A PRIORI ESTIMATES FOR DEGENERATE MONGE-AMPÈRE EQUATIONS

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1. Introduction. In this paper, we are concerned with the Dirichlet problem for degenerate Monge-Ampère equations:

$$\begin{cases} \det(u_{ij}) = f & \text{in } \Omega \subset \mathbf{R}^n \\ u|_{\partial\Omega} = \phi. \end{cases} \tag{1}$$

The Dirichlet problem for elliptic Monge-Ampère equations has received considerable study. It is quite complete due to the important works of Caffarelli-Nirenberg-Spruck [CNS1] and Krylov [K1]. (See also references in the above papers for the history of the problem.) When f is only assumed nonnegative, the equation is degenerate. In this case, in general, one can only expect the solution to be $C^{1,1}$. Thus, much attention has been directed to obtaining C^2 a priori estimates of the problem. The interior and global C^2 estimates were obtained, respectively, by Trudinger-Urbas [TU2] and Caffarelli-Nirenberg-Spruck [CNS2] for the homogeneous equation:

$$\begin{cases} \det(u_{ij}) = 0 & \text{in } \Omega \subset \mathbf{R}^n \\ u|_{\partial\Omega} = \phi. \end{cases} \tag{2}$$

$C^{1,1}$ regularity for inhomogeneous degenerate equations (1) was studied by Caffarelli-Kohn-Nirenberg-Spruck [CKNS] and Krylov [K2] (see also the references therein) under the assumption that $f = g^n$ for some $C^{1,1}$ function g. Since $F(u_{ij}) = \det(u_{ij})^{1/n}$ is concave, the assumption $f = g^n$ seems natural for obtaining C^2 estimates for the equation (1). This assumption has been used in most of the works related to the regularity problem of the equation (1). Except in [C], Caffarelli obtained $C^{1,\alpha}$ regularity for convex solutions of equation (1) under a very general condition that the right-hand side of (1) is a measure satisfying the doubling property. As we know, the assumption $f = g^n$ is quite restrictive. Even in the $n = 2$ case, not every smooth function is a square of a $C^{1,1}$ function. In dealing with geometric problems, one would like to know $C^{1,1}$ (and better) regularity of the solution when f is only assumed to be nonnegative and smooth (e.g., [GL1]). Therefore, we ask: Is the solution of equation (1) in $C^{1,1}$ if f is nonnegative and smooth?

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