

ON QUANTUM GALOIS THEORY

CHONGYING DONG AND GEOFFREY MASON

1. Introduction. The goals of this present paper are to initiate a program to systematically study and rigorously establish what a physicist might refer to as the “operator content of orbifold models.” To explain what this might mean, and to clarify the title of the paper, we assume that the reader is familiar with the algebraic formulation of 2-dimensional conformal field theory (CFT) in the guise of vertex operator algebras (VOA); see [B], [FLM], and [DM] for more information on this point.

In the paper [DVVV], several ideas are proposed concerning the structure of a holomorphic orbifold. In other words, if V is a holomorphic VOA and if G is a finite group of automorphisms of V , then the sub-VOA V^G of G -invariants is itself a VOA and the subject of [DVVV] is very much concerned with speculation on the nature of the V^G -modules.

It turns out to be more useful—at least for purpose of inductive proofs—to take V to be a *simple* VOA. We will then see that V^G is also simple whenever G is a finite group of automorphisms of V . One consequence of our main results is the following.

THEOREM 1. *Let V be a simple VOA and G a finite and faithful group of automorphisms of V . Assume that G is either abelian or dihedral. Then there is a bijection between the subgroups of G and the sub-VOAs of V which contain V^G defined by the map $H \mapsto V^H$.*

This partially explains the title of the paper. We have no reason to believe that Theorem 1 is not true for any finite group G , and indeed we can prove the analogous result for various classes of groups other than just those listed in Theorem 1.

There are other Galois correspondences which may well be relevant to the present context. Three that come to mind are the theory of G -covering spaces, and Galois theory in the context of von Neumann algebras [J] and Hopf algebras [Mo]. Indeed Iiyori and Yamada, in [IY], have taken an approach to results related to Theorem 1 with von Neumann algebras very much in mind.

Our own approach yields Theorem 1 as essentially a consequence of results

Received 10 May 1995. Revision received 11 October 1995.

Dong’s research supported by National Security Agency grant MDA904-92-H-3099, National Science Foundation grant DMS-9303374, a Regent’s Junior Faculty Fellowship of the University of California, and a research grant from the Committee on Research, University of California, Santa Cruz.

Mason’s research supported by National Science Foundation grant DMS-9122030 and a research grant from the Committee on Research, University of California, Santa Cruz.