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POISSON HOMOGENEOUS SPACES AND LIE ALGEBROIDS ASSOCIATED TO POISSON ACTIONS

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CONTENTS

1. Introduction. This work is motivated by a result of Drinfeld in [Dr2]. Recall [Dr1], [STS] that a *Poisson Lie group* is a Lie group G together with a Poisson structure such that the group multiplication map

$$G \times G \to G$$

is a Poisson map. Given a Poisson Lie group G and a Poisson manifold P, an action

$$\sigma: G \times P \to P$$

of G on P is called a *Poisson action* if the action map σ is a Poisson map. When the action is transitive, we say that P is a *Poisson homogeneous G-space.* Poisson G-spaces are the semiclassical analogs of quantum spaces with quantum group actions. Special cases of Poisson homogeneous G-spaces can be found in [DaSo], [Lu1], [Za].

Let P be a Poisson homogeneous G-space. In [Dr2], Drinfeld shows that corresponding to each $p \in P$, there is a maximal isotropic Lie subalgebra $\mathfrak{l}_p$ of the Lie algebra $\mathfrak{d}$, the double Lie algebra of the tangent Lie bialgebra $(\mathfrak{g}, \mathfrak{g}^*)$ of G. Moreover, for $g \in G$, the two Lie algebras $\mathfrak{l}_p$ and $\mathfrak{l}_{gp}$ are related by $\mathfrak{l}_{gp} = \mathrm{Ad}_g \mathfrak{l}_p$ via the adjoint action of G on $\mathfrak{d}$. In particular, they are isomorphic as Lie algebras.

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