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p-ADIC K-THEORY OF HECKE CHARACTERS OF IMAGINARY QUADRATIC FIELDS AND AN ANALOGUE OF BEILINSON'S CONJECTURES

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0. Introduction. In this paper we combine ideas of Soulé [23] and Deninger [5], [6] to prove a p-adic analogue of Beilinson's conjectures for motives associated to Hecke characters of imaginary quadratic fields.

Let E be an elliptic curve defined over an imaginary quadratic field K with complex multiplication by the ring of integers of K. In [23], Soulé proved the following theorem.

Let p be a prime which splits in K and $l \geqslant 0$ such that $p - 1$ divides neither l, $l + 1$, nor $l + 2$. Then there exists a $\mathbb{Z}_p$-submodule $\mathscr{V}_l \subseteq K_{2l+2}(E, \mathbb{Z}_p)$ and a regulator map r_l: $K_{2l+2}(E, \mathbb{Z}_p) \to \mathbb{Z}_p^2$, such that the index of $r_l(\mathscr{V}_l)$ in $\mathbb{Z}_p^2$ equals n_l, where n_l is the p-adic valuation of the value at $s = -l$ of a p-adic L-series analog to $L(E, s)$.

On the other hand, let φ be a Hecke character of an imaginary quadratic field K of positive weight w. Then Deninger constructed a motive M in $\mathscr{M}_{\mathbb{Q}}(K)$, the category of Chow motives over K with coefficients in $\mathbb{Q}$, such that the L-series of M coincides with the L-series of φ. The motive M arises naturally as a factor of the Grothendieck restriction $\mathscr{R}_{F/K}(h_1(E))^{\otimes w}$ for a CM-elliptic curve E of Shimura type over a finite extension F of K. Then he proved parts of the Beilinson conjectures for M; i.e., he related the leading coefficient of $L(M, -l)$ to a map from $H_{\mathscr{A}}^{w+1}(M, \mathbb{Q}(l + w + 1)) = K_{2l+w+1}(M)_{\mathbb{Q}}^{(l+w+1)}$ to Deligne cohomology.

Here we combine the ideas of both papers to prove a generalization of Soulé's theorem for motives M_Ω attached to Hecke characters of infinity type (a, b) and weight $w = a + b > 0$ in the category $\mathscr{M}_{\mathbb{Z}_p}(K)$ of Chow motives over K with coefficients in $\mathbb{Z}_p$. More precisely, we first prove a Grothendieck-Riemann-Roch theorem for K-groups with coefficients $K_a(X, \mathbb{Z}/p^n)$ and p big enough relative to $\dim X$. Then we show that the functors $K_a(-, \mathbb{Z}/p^n)^{(i)}$ factor through $\mathscr{M}_{\mathbb{Z}_p}$, and finally we prove the following theorem.

THEOREM 0.1. *Let $l \geqslant 0$ and $p > (3[F:K] + 1)w + 2l + 1$ be a prime split in the imaginary quadratic field K, $a + l > 0$ and $b + l > 0$. Then there exists a submodule $\mathscr{V} \subseteq K_{2l+w+1}(M_\Omega, \mathbb{Z}_p)^{(l+w+1)}$ such that the length as an $\mathcal{O}_\Omega$-module of the cokernel of the regulator $R|_{\mathscr{V}}$ restricted to $\mathscr{V}$ equals the valuation of the p-adic*

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