Vol. 86, No. 1 DUKE MATHEMATICAL JOURNAL © 1997

ALGEBRAIC CYCLES ON SHIMURA VARIETIES OF ORTHOGONAL TYPE

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Introduction. The occurrence of intersection numbers of (algebraic) cycles in the Fourier coefficients of modular forms is a now rather familar phenomenon which originates in the work of Hirzebruch and Zagier [13]. Their work was generalized to certain quotients of the complex 2-ball in [15], and then to fairly general arithmetic quotients of the symmetric spaces of the classical groups $O(p,q)$, $U(p,q)$, and $Sp(p,q)$ in [17], [18], and [19]. In these cases, the intersection numbers occur in the Fourier expansions of automorphic forms on the groups $Sp(n)$, $U(n,n)$, or $O^*(2n)$, and the phenomenon is directly linked to the geometric properties of the theta correspondence for a classical dual reductive pair.

More recently, Gross and Keating [8] found intersection numbers occuring in the Fourier coefficients of certain *Eisenstein series* for the Siegel modular group of genus 2. Moreover, their calculations and the studies of the central critical value of the triple product L-function [26], [12], [9] strongly suggest that the Fourier coefficients of the derivative, at the center of symmetry, of the Siegel Eisenstein series of genus 3 involve the height pairings or arithmetic intersection numbers of certain curves on a threefold. The occurrence of heights in such Fourier coefficients has recently been confirmed in certain other cases [16], [22]. Thus it appears that much remains to be understood in this subject.

In this article, we begin the detailed arithmetic study of the algebraic cycles in locally symmetric varieties associated to orthogonal groups $O(p,2)$. These cycles are special cases of those whose intersection numbers are considered in [19], and they are also locally symmetric, associated to orthogonal subgroups $O(p-n,2)$ of $O(p,2)$. Our main results are the following. First we define "weighted" sums of such cycles which behave nicely with respect to the coverings defined by congruence subgroups. These "weighted cycles" are higher-dimensional analogues of the Hirzebruch-Zagier cycles T_N, and they define a sort of modular symbol, whose parameter is a pair consisting of a positive semidefinite rational symmetric matrix β and a weight function φ. The subspace of the Chow group spanned by these cycles as β and φ vary is stable under the Hecke operators, as is its image in any cohomology theory.

Next, it turns out that the methods and results of [17], [18], [19] allow us to give an *explicit description of the cup product* of the Betti cohomology classes

Received 23 February 1995. Revision received 14 March 1996.
Kudla was partially supported by National Science Foundation grant DMS-9302539.