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CONSTANT MEAN CURVATURE SURFACES WITH PLANAR BOUNDARY

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1. Introduction. A constant mean curvature surface immersed in Euclidean 3-space can be viewed as a surface where the exterior pressure and the surface tension forces are balanced. For this reason they are thought of as *soap bubbles or films* depending on the considered surface being either closed (that is, compact without boundary) or compact with nonempty boundary. With respect to the closed case, until 1986 the only known examples of constant mean curvature surfaces were the round spheres. In that year, Wente [W4] constructed genus-1 constant mean curvature surfaces which are nonembedded (see also [Ab, Bo, PS]). One year later, Kapouleas [Ka] did the same for genera bigger than 2. These results activated in a remarkable way the research in this field and indicated the sharpness of the two principal theorems about closed constant mean curvature surfaces which were known at that time: the Hopf theorem, which asserts that the sphere is the only example with genus zero [Ho] and the Alexandrov theorem which says that the sphere is the only possible embedded example [Al].

With respect to the study of the space of compact constant mean curvature H surfaces with prescribed nonempty boundary Γ, we do not know its structure even in the easiest case, when Γ is a round circle with, for instance, unit radius. Heinz [He2] found that a necessary condition for existence in this situation is that $|H| \leq 1$. The only known examples, excluding the trivial minimal case, are the two spherical caps with radius $1/|H|$, which are the only umbilical ones, and some nonembedded surfaces of genus bigger than 2 whose existence was proved by Kapouleas in [Ka].

In general, when Γ is a Jordan curve in $\mathbb{R}^3$, the problem of existence of constant mean curvature H surfaces Σ with $\partial\Sigma = \Gamma$ has been studied by Heinz [He1], Hildebrant [Hi], Wente [W1], Werner [We], and Steffen [St] in the case of immersions from the 2-dimensional disc. They solved the corresponding (disc-parametric) Plateau problem (when H is small enough in terms of the geometry of the curve Γ) by showing existence of *small* (that is, contained in some sphere with radius less than $1/|H|$) solutions which are relative minimizers of the functional $A - 2HV$, A being the area and V the algebraic volume functionals, respectively. On the other hand, Serrin [Se1] proved, using continuity methods, the existence of constant mean curvature graphs on some strictly convex planar domains. All these works have culminated in those of Brézis and Coron [BrC]

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