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# ON POSITIVITY, CRITICALITY, AND THE SPECTRAL RADIUS OF THE SHUTTLE OPERATOR FOR ELLIPTIC OPERATORS

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**1. Introduction.** Let $X$ be a smooth, noncompact, connected manifold, and consider a second-order linear elliptic operator $P$ defined on $X$. Let $\Omega$ be a domain in $X$, and denote by $\mathscr{C}_P(\Omega)$ the convex cone of all classical, positive solutions of the equation

$$Pu = 0 \qquad \text{in } \Omega. \tag{1.1}$$

We recall ([11], [14]; see also Section 2 for a more detailed definition) that $P$ is a *subcritical operator* in $\Omega$ if it possesses a positive minimal Green function $G_P^\Omega(x, y)$ in $\Omega$. The operator $P$ is said to be *critical* in $\Omega$ if it is not subcritical, but $\mathscr{C}_P(\Omega) \neq \emptyset$. Finally, $P$ is *supercritical* if $\mathscr{C}_P(\Omega) = \emptyset$.

Given an operator $P$ and a domain $\Omega$, there are several criteria to distinguish between the above three situations. These criteria depend on the behavior of the cone of positive solutions under perturbations of either the operator $P$ or the domain $\Omega$ (see, for example, [5], [6], [9], [10], [11], [14], [17], and the references therein). Thus, they are of perturbation nature. But, so far, there is no general intrinsic criterion depending only on the given operator $P$ in $\Omega$, which distinguishes between the three possibilities. Recall also that we do not impose any boundary behavior at "infinity" for solutions in $\mathscr{C}_P(\Omega)$. Therefore, the natural topology in the theory of positive solutions is the compact open topology and, in general (especially in the nonselfadjoint case), we usually do not have any information in Banach spaces' terminology.

The aim of this paper is to present an intrinsic criterion, which distinguishes between subcriticality, criticality, and supercriticality of the operator $P$ in $X$. Moreover, this criterion depends only on the norm of a certain linear operator defined on a Banach space. More precisely, we introduce and study a linear operator $S$ associated with the differential operator $P$ in $X$. The operator $S$ is called the *shuttle operator* and is defined on $C(\partial D)$, the Banach space of continuous functions on the boundary of a certain smooth, relatively compact subdomain $D$. It turns out (Theorem 4.1) that the norm and the spectral radius of

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