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TORSION ZERO-CYCLES ON THE SELF-PRODUCT OF A MODULAR ELLIPTIC CURVE

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§0. Introduction. Let E be a modular elliptic curve defined over $\mathbb{Q}$ with the conductor N, and let $X = E \times_{\mathbb{Q}} E$, which is by definition a projective smooth surface over $\mathbb{Q}$. Let $CH_0(X)$ be the Chow group of zero-cycles on X modulo rational equivalence. Fix a prime p and let

$$\rho_p : CH_0(X)\{p\} \to H^4_{\text{cont}}(X, \mathbb{Z}_p(2))$$

be the cycle map, where $CH_0(X)\{p\} \subset CH_0(X)$ is the subgroup of the p-primary torsion elements and the group on the right-hand side is the continuous p-adic etale cohomology group (cf. [J1]). The main result of the paper is the following.

THEOREM 0-1. *Assume E has no complex multiplication over any finite extension of $\mathbb{Q}$ and that N is square-free and $p \nmid 6N$. Then ρ_p is injective.*

We remark that the assumption $p \nmid 6$ is due to a certain technical problem in the p-adic Hodge theory (cf. Theorem 6-1). The assumption $p \nmid N$ is more essential.

As a corollary of Theorem 0-1, one obtains the following.

THEOREM 0-2. *Let the assumption be as Theorem 0-1. Then $CH_0(X)\{p\}$ is finite.*

Indeed, take a proper smooth model $\mathscr{X}$ of X over a nonempty open subscheme of $\mathrm{Spec}(\mathbb{Z}[1/p])$. The argument of the proof of [Sa3, (4-4)] shows that the image of ρ_p is contained in the image of the natural map

$$H^4_{\text{et}}(\mathscr{X}, \mathbb{Z}_p(2)) \to H^4_{\text{cont}}(X, \mathbb{Z}_p(2)),$$

which is a finitely generated $\mathbb{Z}_p$-module.

We have the following additional result.

THEOREM 0-3. *There exists a finite set S of rational primes for which we have*

$$CH_0(X)\{p\} = 0 \qquad \text{if } p \notin S.$$

(See the appendix for a more precise statement.)

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