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LEFSCHETZ CLASS OF ELLIPTIC PAIRS

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1. Introduction. In [10] P. Schapira and J.-P. Schneiders introduced the notion of elliptic pairs, generalizing the notion of ellipticity in the following way: An elliptic pair on a complex manifold X is the data of a real constructible sheaf F and a coherent $\mathscr{D}_X$-module $\mathscr{M}$ which satisfy

$$SS(F) \cap \operatorname{char}(\mathscr{M}) \subset T_X^*X,$$

where $SS(F)$ stands for the microsupport of F (see [8]) and $\operatorname{char}(\mathscr{M})$ for the characteristic variety of $\mathscr{M}$. For a morphism of complex manifolds $f: X \to Y$, they also introduced a notion of relative characteristic variety (denoted $\operatorname{char}_f(\mathscr{M})$) and relative elliptic pair (substituting $\operatorname{char}_f(\mathscr{M})$ for $\operatorname{char}(\mathscr{M})$ in the preceding inclusion). Like elliptic systems, elliptic pairs satisfy an important finiteness property: they showed that the direct image of a relative elliptic pair, $\underline{f}_!(F \otimes \mathscr{M})$, is a coherent $\mathscr{D}_Y$-module. Defining the dual of an elliptic pair $(F, \mathscr{M})$ to be the pair of the duals of its components $(D'F, \underline{D}\mathscr{M})$, they proved the following duality result: duality commutes with direct image. They showed other results and among them a Künneth formula.

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