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LAGRANGIAN INTERSECTION UNDER LEGENDRIAN DEFORMATIONS

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1. Introduction. Lagrangian submanifolds play an important role in symplectic geometry and have special features. For instance, a middle-dimensional submanifold in a symplectic manifold which is not lagrangian can be separated from itself by a small hamiltonian isotopy, provided it is possible in differentiable categories [LS2]. But this is not the case for lagrangian submanifolds. For a small hamiltonian isotopy, the problem reduces to the case of the cotangent bundle of the lagrangian submanifold (by the symplectic neighborhood theorem) and persistence of intersection points; this was established by Hofer [H], and Laudenbach and Sikorav [LS1]. For a general hamiltonian isotopy, we have the following.

Question (Arnold's conjecture for lagrangian submanifolds). Let L be a lagrangian submanifold in a symplectic manifold (M, ω). For a hamiltonian isotopy $\{\phi_t | 0 \leqslant t \leqslant 1\}$, the number of intersection points of L and $\phi_1(L)$ is at least the cup-length of L. Moreover, if they intersect transversally, this number is at least the sum of Betti numbers of L.

The result of Hofer, and Laudenbach and Sikorav mentioned above answered this question affirmatively in the case of the zero section of the cotangent bundle of a compact manifold. In general, we need more conditions to show this statement. (For instance, it is not true for a circle in the complex plane.)

Floer initiated an analog of Morse theory for lagrangian intersections and verified the statement above in the case that M is a closed manifold and $\pi_2(M, L) = 0$ [F]. Specifically, he introduced the so-called Floer homology group for lagrangian pairs and showed that this group is isomorphic to the homology group of L under the condition above. The construction of Floer homology for lagrangian pairs was generalized by Oh in the class of monotone lagrangian submanifolds with minimal Maslov number being at least 3 [Oh]. However, computation of these groups is a subtler problem than the case of Floer homology for periodic hamiltonian systems and has not been developed as much, so far, as the periodic Hamiltonian case [HS], [On].

In this paper, we shall discuss a similar problem for "legendrian deformations" of lagrangian submanifolds. Suppose that the symplectic form ω represents an

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