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HIGHER CHOW GROUPS AND THE HODGE-$\mathscr{D}$-CONJECTURE

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0. Introduction. Let X be a complex, quasi-projective variety of dimension n, with projective closure $\overline{X}$, and let $Y = \overline{X} - X$. Define $CH_{*+m}(-) = \bigoplus_{k\geqslant 0} CH_{k+m}(-)$, and $W_{-*}H_{*+m}(-) = \bigoplus_{i\geqslant 0} W_{-i}H_{i+m}(-)$, $W_{\cdot}$ = weight filtration. In terms of describing the influence of mixed Hodge structures on Chow groups, there is the following schema below (with exact rows) (note: All homology is Borel-Moore, with $\mathbf{Q}$-coefficients):

$$\begin{array}{ccccccccccc}
\cdots \to W_{-*}H_{*+m}(X) & \to \cdots \to & W_{-*}H_{*+1}(X) & \to & W_{-*}H_*(Y) & \to & W_{-*}H_*(\overline{X}) & \to & W_{-*}H_*(X) & \to 0 \\
\updownarrow ? & & \updownarrow ? & & \updownarrow & & \updownarrow & & \updownarrow & \\
\cdots \to CH_{*+m}(X,m) & \to \cdots \to & CH_{*+1}(X,1) & \to & CH_*(Y,0) & \to & CH_*(\overline{X},0) & \to & CH_*(X,0) & \to 0
\end{array}$$

where $CH_{\dim Z+m-i}(Z,m) \stackrel{\text{def}}{=} CH^i(Z,m)$ are the higher Chow groups introduced by Bloch [B1], and where $CH_*(-,0) = CH_*(-)$ (as defined in [F]). The story for $m = 0$ was worked out in [L1]. We would like to speculate about a possible relationship between $W_{-*}H_{*+m}(X)$ and $CH_*(X,m)$ as a generalization of the case $m = 0$.

Let $H = H_{\mathbf{Q}}$ be a Hodge structure, with Hodge decomposition $H_{\mathbf{C}} = \bigoplus_{p,q} H^{p,q}$. We define $\mathrm{Level}(H) = \max\{p - q \mid H^{p,q} \neq 0\}$ if $H \neq 0$ and $\mathrm{Level}(H) = -\infty$ if $H = 0$. We also define $F^k_{\mathbf{Q}}H$ to be the maximum Hodge structure contained in $H_{\mathbf{Q}} \cap F^kH_{\mathbf{C}}$. As an initial guess based on earlier work [L1] and the above schema, it seems reasonable to expect that $Gr_{-k-\ell}F^{\cdot}_{\mathbf{Q}}\{\bigoplus_{i\geqslant 2k+\ell} Gr_{-i}W_{\cdot}H_{2k+\ell+m}(X,\mathbf{Q})\}$ influences $CH_{k+m}(X,m)$ to some degree. The range of ℓ considered is $m \leqslant \ell \leqslant n-k$.

For $k < m$, we define $\mathrm{Level}(CH^k(X,m)) = 0$. Otherwise, for $k \geqslant m$, we set $\mathrm{Level}(CH^k(X,m)) :=$

$$\min\{r \geqslant 0 \mid CH^k(X,m) \to CH^k(X-Y,m) \text{ is zero},$$

$$Y \hookrightarrow X \text{ closed, } \mathrm{codim}_X Y = k - r - m\}.$$

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