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LOCAL STATISTICS FOR RANDOM DOMINO TILINGS OF THE AZTEC DIAMOND

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1.1. Summary. We prove an asymptotic formula for the probability that, if one chooses a domino tiling of a large Aztec diamond at random according to the uniform distribution on such tilings, the tiling will contain a domino covering a given pair of adjacent lattice squares. This formula quantifies the effect of the diamond's boundary conditions on the behavior of typical tilings; in addition, it yields a new proof of the arctic circle theorem of Jockusch, Propp, and Shor. Our approach is to use the saddle point method to estimate certain weighted sums of squares of Krawtchouk polynomials (whose relevance to domino tilings is demonstrated elsewhere), and to combine these estimates with some exponential sum bounds to deduce our final result. This approach generalizes straightforwardly to the case in which the probability distribution on the set of tilings incorporates bias favoring horizontal over vertical tiles or vice versa. We also prove a fairly general large deviation estimate for domino tilings of simply connected planar regions that implies that some of our results on Aztec diamonds apply to many other similar regions as well.

1. Introduction

1.2. Statement of the main theorem. Random domino tilings of finite regions often exhibit surprising statistical heterogeneity. Such heterogeneity would be expected in the vicinity of the boundary, but in fact the presence of a boundary can make its influence felt well into the interior of the region. The research that led to this article is part of an ongoing effort to understand this phenomenon. The results proved here are the first to give a precise description of how local statistics for domino tilings can vary continuously throughout a region in response to the imposition of specific boundary conditions.

Those who study random tilings of finite regions (in the plane) by dominos have tended to focus on regions that are rectangles of even area. In particular, Burton and Pemantle [BP] have done an intensive analysis of the small-scale structure of such tilings. Their work shows that once one gets away from the

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