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GLOBAL BIFURCATION RESULTS FOR A SEMILINEAR ELLIPTIC EQUATION ON ALL OF $\mathbb{R}^N$

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1. Introduction. In this paper we shall discuss the existence of positive solutions of the equation

$$-\Delta u(x) = \lambda g(x) f(u(x)), \qquad x \in \mathbb{R}^N, \tag{1.1$_\lambda$}$$

$$0 < u < 1, \qquad x \in \mathbb{R}^N, \qquad \lim_{|x| \to +\infty} u(x) = 0, \tag{1.2}$$

by using methods of bifurcation theory. The equation arises in population genetics (see [6]) where the function g is assumed to change sign and $f\colon [0,1] \to \mathbb{R}^+$, with $f(0) = f(1) = 0$. The unknown function u corresponds to the relative frequency of an allele and is hence constrained to have values between 0 and 1. The real parameter $\lambda > 0$ corresponds to the reciprocal of a diffusion coefficient.

The problem is well understood on bounded domains where a fairly complete bifurcation analysis can be given (see [3]). The situation is more complicated in the case of unbounded domains as, in general, the equation does not give rise to compact operators, and so it is unclear that there exist eigenvalues from which bifurcation can occur. It is also unclear a priori in which function spaces solutions of $(1.1)_\lambda$ might lie.

To discuss bifurcation from the zero solution of $(1.1)_\lambda$, it is first necessary to study the eigenvalues of the corresponding linear problem

$$\begin{aligned} -\Delta u(x) &= \lambda g(x) f'(0)\, u(x) \qquad \text{for } x \in \mathbb{R}^N \\ \lim_{|x| \to +\infty} u(x) &= 0. \end{aligned} \tag{1.3}$$

The existence of a positive principal eigenvalue (i.e., an eigenvalue corresponding to a positive eigenfunction and so a point at which positive solutions of $(1.1)_\lambda$ may bifurcate from the zero branch) for the above problem has been proved in [2], [4] under the hypotheses that $\int_{\mathbb{R}^N} g(x)\, dx < 0$ and $g(x) < 0$ for $|x|$ large, and in [1] under the hypothesis that $N \geqslant 3$ and $g_+ \in L^{N/2}(\mathbb{R}^N)$.

We shall discuss the local and global bifurcation of solutions of $(1.1)_\lambda$ in the

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