Vol. 84, No. 3 DUKE MATHEMATICAL JOURNAL © September 1996

MULTIPEAK SOLUTIONS FOR A SEMILINEAR NEUMANN PROBLEM

CHANGFENG GUI

1. Introduction. In a series of remarkable papers, W.-M. Ni and I. Takagi studied the Neumann problem for certain semilinear elliptic equations, including

$$\begin{cases} d\Delta u - u + u^p = 0, \quad u > 0 \quad \text{in } \Omega, \\ \partial u/\partial \nu = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded smooth domain in $\mathbf{R}^N$, $N \geqslant 2$, and ν is the unit outer normal to $\partial\Omega$; $d > 0$, $p > 1$ are constants; and p is subcritical, i.e., $p < (N+2)/(N-2)$. (It is understood as $p < \infty$ when $N = 2$.) First, Ni, Takagi, and C. Lin [LNT] applied the Mountain Pass Lemma to show the existence of a least-energy solution u_d to (1.1), by which it is meant that u_d has the smallest energy among all the solutions to (1.1), with the energy functional being

$$I_d(u) = \int_\Omega \left(\frac{d}{2} |\nabla u|^2 + \frac{1}{2} u^2 - \frac{1}{p+1} u_+^{p+1} \right) dx, \tag{1.2}$$

where $u_+ = \max\{u, 0\}$. Then, in [NT1], [NT2], Ni and Takagi investigated the shape of the least-energy solution u_d for d sufficiently small and showed that u_d has exactly one peak (i.e., local maximum of u_d) at P_d. Moreover, P_d is on the boundary of Ω, and as d tends to 0, P_d approaches a point at which the mean curvature of $\partial\Omega$ is the maximum.

In this paper, we are concerned with solutions to (1.1) which have higher energy than that of u_d. In particular, we obtain solutions with single peaks as well as solutions with multiple peaks, with all of the peaks being on the boundary and located near local maxima of the mean curvature of $\partial\Omega$.

To be more precise, let Ω be a bounded smooth domain. We consider the following general semilinear Neumann problem:

$$\begin{cases} \varepsilon^2\Delta u - u + f(u) = 0, \quad u > 0 \quad \text{in } \Omega, \\ \partial u/\partial \nu = 0 \quad \text{on } \partial\Omega. \end{cases} \tag{1.3}$$

Received 27 June 1995. Revision received 15 January 1996.