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TWISTED S-UNITS, p-ADIC CLASS NUMBER FORMULAS, AND THE LICHTENBAUM CONJECTURES

MANFRED KOLSTER, THONG NGUYEN QUANG DO, AND VINCENT FLECKINGER

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0. Introduction. Let F be a number field with ring of integers O_F. For a given *odd* rational prime p let $S = S(F)$ denote the set of primes of F above p. Denote by Ω_S the maximal algebraic S-ramified extension of F and set $G_S = G_S(F) = \mathrm{Gal}(\Omega_S/F)$. It is well known that the p-adic cohomology groups $H^i(G_S(F), \mathbb{Z}_p(m)) = \varprojlim H^i(G_S(F), \mathbb{Z}/p^n\mathbb{Z}(m)), m \in \mathbb{Z}$, coincide with the étale cohomology groups $H^i_{\text{ét}}(\operatorname{spec} O_F[1/p], \mathbb{Z}_p(m))$, which we simply denote by $H^i(O_F, \mathbb{Z}_p(m))$. This paper is mainly devoted to the study of the cohomology groups $H^1(O_F, \mathbb{Z}_p(m))$, $m \in \mathbb{Z}$, of some of their interesting subgroups, and of canonically attached p-adic regulators. These results are then used to prove the Lichtenbaum Conjectures for abelian fields.

In a sense this could be considered as a continuation of P. Schneider's paper [35] on the Galois cohomology groups $H^i(G_S, \mathbb{Q}_p/\mathbb{Z}_p(m))$, $m \in \mathbb{Z}$, and as a revisitation of Soulé's papers [39], [40] on higher p-adic regulators in K-theory. It seems therefore appropriate to recapitulate some of the known results and to give an overview of the new ones.

0.1. The interest in the groups $H^1(O_F, \mathbb{Z}_p(m))$ comes from the fact that they give a unified description, where m varies over $\mathbb{Z}$, of seemingly unrelated arithmetical objects associated with the pair (F, p).

(i) For $m = 0$, the group $H^1(O_F, \mathbb{Z}_p) = \mathrm{Hom}(G_S(F), \mathbb{Z}_p)$ classifies $\mathbb{Z}_p$-extensions (in the sense of Galois algebras) of F and has been studied in [10] in relation with the weak Leopoldt Conjecture (see also [11]).

(ii) For $m = 1$, Kummer theory immediately shows that $H^1(O_F, \mathbb{Z}_p(1)) \cong$

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