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SHEAVES WITH CONNECTION ON ABELIAN VARIETIES

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1. Introduction. Let X and Y be abelian varieties over an algebraically closed field k, dual to one another, and let $\mathrm{Mod}(\mathcal{O}_X)$ and $\mathrm{Mod}(\mathcal{O}_Y)$ be their respective categories of quasi-coherent $\mathcal{O}$-modules. Mukai proved in [Muk] that the derived categories $D\,\mathrm{Mod}(\mathcal{O}_X)$ and $D\,\mathrm{Mod}(\mathcal{O}_Y)$ are equivalent via a transform now known as the Fourier-Mukai transform,

$$\mathcal{S}_1(\mathcal{F}) = \alpha_{2*}(\mathcal{P} \otimes \alpha_1^*(-1)^*(\mathcal{F})), \tag{1.1}$$

where $\mathcal{P}$ is the Poincaré sheaf and α_1 and α_2 are the projections from $X \times Y$ to X and Y, respectively. A few years earlier, Krichever [K] rediscovered a construction due originally to Burchnall and Chaundy [BC], by which the affine coordinate ring of a projective curve minus a point may be imbedded in the ring of formal differential operators in one variable. The construction involves the choice of a line bundle on the curve, and Krichever took the crucial step of asking, in the case of a smooth curve, how the imbedding varies when the line bundle moves linearly on the Jacobian. The answer is now well known, that the imbeddings satisfy the system of differential equations known as the KP-hierarchy. In fact, the Krichever construction is an instance of the Fourier-Mukai transform, with the crucial addition that the transformed sheaf is not only an $\mathcal{O}_Y$-module but a $\mathcal{D}_Y$-module, where $\mathcal{D}_Y$ is the sheaf of linear differential operators on Y; see [N1], [N2], [R].

This example serves as the inspiration for the present work, which is concerned with the role of the Fourier-Mukai transform in the theory of sheaves on Y equipped with a connection. The main point is that in the derived category, all sheaves on Y with connection are constructed by the Fourier-Mukai transform in a manner directly generalizing the Krichever construction. The connection need not be integrable, though the paper focuses mostly on that case.

The basic idea is the following. Set

$$\mathfrak{g} = H^1(X, \mathcal{O}). \tag{1.2}$$

Then there is a tautological extension

$$0 \longrightarrow \mathfrak{g}^* \otimes \mathcal{O} \longrightarrow \mathcal{E} \xrightarrow{\mu} \mathcal{O} \longrightarrow 0 \tag{1.3}$$

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