

THE NIELSEN-THURSTON CLASSIFICATION AND AUTOMORPHISMS OF A FREE GROUP I

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Using train tracks and invariant laminations, Thurston has developed a whole theory to understand the dynamics and geometry of diffeomorphisms of surfaces [Th], [CB]. By introducing a combinatorial analogue of train tracks, M. Bestvina and M. Handel [BH] have managed to analyze irreducible automorphisms of a free group and, using this analysis, to bound the rank of the fixed subgroup of an automorphism by the rank of the ambient group, which was known before as the Scott conjecture.

In [Se3], borrowing Jaco-Shalen-Johannson's (JSJ) theory of the characteristic submanifold, we introduced a canonical decomposition for freely indecomposable (Gromov) hyperbolic groups, which serves as a fundamental object for generalizing results from the mapping class group of surfaces to automorphisms and the automorphism group of freely indecomposable hyperbolic groups, and in particular generalized Thurston's work to this class of groups. This JSJ decomposition was later generalized to finitely presented (f.p.) groups in [RS].

The construction of the JSJ decomposition, which extensively uses Rips's work on real trees, succeeds in generalizing Thurston's theory, but has not been able to suggest an alternative approach to Thurston's original work. In addition, the whole construction relies extensively on the groups in question being freely indecomposable and above all having no free factors.

In this sequence of three papers (see [Se4]), we construct a hierarchical decomposition of a free group with respect to a given automorphism of it. Our construction, which is dynamical in nature, suggests a "unified" approach to Thurston's theory on the dynamics of diffeomorphisms of surfaces and to the study of the dynamics of automorphisms of a free group, and may sometimes serve as a complement to the Bestvina-Handel train tracks and invariant laminations. We also believe that, besides the applicability of our methods to the study of the dynamics of automorphisms of a free group, it can be applied in various other cases. In particular we have already used it to obtain the Hopf property for hyperbolic groups [Se2].

In this paper, we introduce the basic tools needed for the construction of the hierarchical decomposition. We start by introducing a dynamical-algebraic commutative diagram, which is the basis for our whole approach. This commu-

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