

A UNIVERSAL MULTICOEFFICIENT THEOREM FOR THE KASPAROV GROUPS

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1. Introduction. Let $\underline{K}(A)$ denote the sum of all the K -theory groups of a C^* -algebra A in all degrees and with all cyclic coefficient groups. The Bockstein operations (which generate a category Λ) act on $\underline{K}(A)$. We establish a universal coefficient exact sequence

$$0 \rightarrow \text{Pext}(K_*(A), K_*(B)) \xrightarrow{\delta} KK(A, B) \xrightarrow{\Gamma} \text{Hom}_\Lambda(\underline{K}(A), \underline{K}(B)) \rightarrow 0$$

that holds in the same generality as the universal coefficient theorem of Rosenberg and Schochet.

There are advantages, in some circumstances, to using $\text{Hom}_\Lambda(\underline{K}(A), \underline{K}(B))$ in place of $KK(A, B)$. These advantages derive from the fact that $\underline{K}(A)$ can be equipped with order and scale structures similar to those on $K_0(A)$. With this additional structure, the Λ -module $\underline{K}(A)$ becomes a powerful invariant of C^* -algebras. We show that it is a complete invariant for the class of real-rank-zero AD algebras. The AD algebras are a certain kind of approximately sub-homogeneous C^* -algebras which may have torsion in K_1 [Ell]. In addition to classifying these algebras, we calculate their automorphism groups up to approximately inner-automorphisms.

1.1. Summary. The universal coefficient theorem (UCT) of Rosenberg and Schochet [RS] states the existence of an exact sequence

$$0 \rightarrow \text{Ext}_{\mathbb{Z}}^1(K_*(A), K_*(B)) \xrightarrow{\delta} KK_*(A, B) \xrightarrow{\gamma} \text{Hom}(K_*(A), K_*(B)) \rightarrow 0$$

for many pairs of C^* -algebras A and B . Loosely speaking, this shows that $KK_*(A, B)$ can be regarded as the K -theory of B with coefficients in $K^*(A)$. In particular, it shows that the coefficient group \mathbb{Z} is universal in the sense that, knowing $K_*(B; \mathbb{Z}) = K_*(B)$, we can determine the K -theory for B with other coefficients. For example,

$$K_*(B; \mathbb{Z}/p) = KK_*(A_p, B),$$

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