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AUTOMORPHIC FUNCTIONS ON GENERAL DOMAINS IN $\mathbf{C}^n$

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Except in the special case of symmetric domains little is known about automorphic functions on domains in $\mathbf{C}^n$. Very roughly one would like conditions which guarantee the existence of "enough" automorphic functions and conditions which prohibit the existence of "too many" automorphic functions. The prototypical results of this type (see [5] and [6]) are the following.

Let $U \subset \mathbf{C}^n$ be connected, and suppose Γ acts properly discontinuously biholomorphically on U. Let $K(\Gamma, U)$ denote the field of Γ-automorphic functions on U, i.e., Γ-invariant meromorphic functions on U.

THEOREM 1. *If U is bounded then $K(\Gamma, U)$ has transcendence degree at least n.*

THEOREM 2. *If $\Gamma\backslash U$ is compact then $K(\Gamma, U)$ has transcendence degree at most n.*

Various authors have given differential geometric conditions on $\Gamma\backslash U$ which lead to theorems of type 1 and 2; see [7] for references. Section 8 contains a proof of the following generalization of Theorem 1.

THEOREM 3. *If there is some $\delta > 0$ such that*

$$\int_U (1 + |z|^2)^\delta < \infty, \tag{1}$$

then $K(\Gamma, U)$ has transcendence degree at least n.

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