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LAPLACE TRANSFORMATION IN HIGHER DIMENSIONS

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1. Introduction. The study of the deep relationship between differential geometry and partial differential equations has a long and distinguished history, going back to the works of Darboux, Lie, Bäcklund, Goursat, and E. Cartan. This relationship stems from the fact that most of the local properties of surfaces are expressed naturally in terms of partial differential equations. For example, the condition that a graph $z = z(x, y)$ having constant Gaussian curvature gives rise to a Monge-Ampère equation for $z(x, y)$. Equivalently, the same condition is expressed by the sine-Gordon equation for negative curvature or the elliptic sinh-Gordon equation for positive curvature.

It is therefore very important to study the transformations of surfaces that preserve geometric properties expressible as partial differential equations, since the analytic formulation of these transformations will give rise to mappings preserving the class of partial differential equations under consideration. Ultimately, one expects the geometry of the transformed surface to have considerable implications at the analytic level, notably in terms of the explicit integrability of the underlying partial differential equations.

Perhaps the best-known example of such a transformation of surfaces is Bäcklund's transformation, which takes a surface of constant negative Gaussian curvature (a pseudospherical surface) into another such surface. Since pseudospherical surfaces correspond to solutions of the sine-Gordon equation, the analytic formulation of this transformation will define a mapping, taking solutions of the sine-Gordon equation into other solutions. Bäcklund transformations have become a very important ingredient in the theory of solitons solutions of completely integrable equations.

An equally interesting, but perhaps lesser-known, example of a transformation of surfaces is the Laplace transformation. One starts from a surface S admitting a net of curves which is conjugate for the second fundamental form. Taking the net to be the parametric net with parameters u and v, one has

$$X_{uv} = \Gamma_{12}^1 X_u + \Gamma_{12}^2 X_v, \tag{1}$$

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