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ROITMAN'S THEOREM FOR SINGULAR COMPLEX PROJECTIVE SURFACES

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Introduction. If X is a smooth projective surface over the complex numbers **C**, the classical Abel-Jacobi map goes from the Chow group $A_0(X)$ of cycles of degree zero to the (group underlying the) Albanese variety Alb(X). Roitman's theorem [36] states that this map induces an isomorphism on torsion subgroups. (See [10] for a nice compendium).

The goal of this paper is to remove the word "smooth" from Roitman's theorem. For this we shall modify the definition of $A_0(X)$, replace Alb(X) with Griffiths's intermediate Jacobian $J^2(X)$, and construct a generalization of the Abel-Jacobi map.

MAIN THEOREM. *Let X be a reduced projective surface over* **C**. *Then there is a natural map from $A_0(X)$ to $J^2(X)$ inducing an isomorphism on torsion:*

$$A_0(X)_{\text{tors}} \cong J^2(X)_{\text{tors}}.$$

In particular, the torsion subgroup is a finite direct sum of copies of **Q**/**Z**.

If X is a normal surface, this theorem is a reformulation of a theorem of Collino and Levine [9], [25], because (as we will show in Corollary 4.3), $J^2(X)$ is isomorphic to the Albanese of any desingularization of X.

Gillet studied the Abel-Jacobi map in [19] when X is a singular surface with "ordinary multiple curves" (e.g., a seminormal surface with smooth normalization $\tilde{X}$). He proved in [19, Theorem B] that if $\tilde{X}$ satisfied some extra hypotheses ($p_g = 0$, etc.), then the Abel-Jacobi map is surjective with finite kernel. Thus we deduce the following.

COROLLARY. *Let X be a surface with ordinary multiple curves such that $H^2(X, \mathcal{O}_X) = 0$. Assume that Bloch's conjecture holds for the normalization $\tilde{X}$ of X. Then the Abel-Jacobi map is an isomorphism*

$$A_0(X) \cong J^2(X).$$

We now describe the ingredients in our Main Theorem. If X is a proper surface

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