ON A-INTEGRABILITY OF THE SPECTRAL SHIFT FUNCTION OF UNITARY OPERATORS ARISING IN THE LAX-PHILLIPS SCATTERING THEORY

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1. **Introduction.**   The spectral shift function (s.s.f.) first appeared in the work of theoretical physicist I. M. Lifshitz [20] in 1952 in connection with the quantum theory of crystals. The importance of this object drew the attention of M. G. Krein ("s.s.f." is his term), who constructed a general theory of the s.s.f. [16], [17], [18]. Namely, it was proved that for a pair of unitary operators U, U_0, differing by a trace-class operator, there exists a unique (up to an additive constant) real-valued summable function $\eta(\xi)$ (s.s.f.) such that the trace formula

$$tr\{\varphi(U) - \varphi(U_0)\} = \int_{|\xi|=1} \eta(\xi)\, d\varphi(\xi) \qquad (1.1)$$

holds, φ being an arbitrary function of some suitable class.

M. G. Krein and M. Sh. Birman [5] established a fundamental relationship between the s.s.f. and the scattering matrix S_ξ of the pair (U, U_0):

$$e^{-2\pi i \eta(\xi)} = \det S_\xi, \qquad \text{a.e. } \xi, |\xi| = 1. \qquad (1.2)$$

A detailed exposition of the mathematical theory of the s.s.f. can be found in a recent survey [6] and book [31].

The works [5], [16], [17], and [18] were followed by numerous papers with the key words "trace formulae," "spectral shift function," "perturbation determinant," and "spectral identities." Moreover, interest in these objects is rather stable, due to their new applications in theoretical and mathematical physics. First of all, we should mention some recent applications to the research on the regularized Witten index in supersymmetric quantum mechanics: [7], [8], [12]; see also [6] and [31] and the literature cited there.

There is also literature devoted to various generalizations of (1.1). Some references can be found in [6] and [31]. We mention here only those that are related to the present work.

Koplienko shows in [14] that the condition that $U - U_0$ is of trace class can be a bit weakened if we are interested in the local existence of the s.s.f. In [15], he considers the case when $U - U_0$ is only of the Hilbert-Schmidt class. Formula