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TOPOLOGICAL SIGMA MODEL AND DONALDSON-TYPE INVARIANTS IN GROMOV THEORY

YONGBIN RUAN

1. Introduction. A symplectic manifold is a pair (V, ω) where V is a smooth manifold of dimension $2n$ and ω is a nondegenerate closed 2-form. The obvious topological invariant is the period $[\omega]$, the cohomology class of the symplectic form ω. Every symplectic manifold admits a family of natural, tamed, almost complex structures, and so it has a canonical orientation. The homotopy class of tamed, almost complex structures is also a topological invariant. Contrary to intuition, the flexibility of symplectic structures makes symplectic geometry rather like a topological theory. For example, let us consider all the hypersurfaces of fixed degree in projective space of a given dimension. Each carries a different Kahler structure from the projective space. If we forget the Kahler structure and only view the hypersurfaces as symplectic manifolds, all of them have the same symplectic structure up to diffeomorphisms by Moser's theorem. More evidence of the topological nature of symplectic geometry is provided by Darboux's theorem that there is no local symplectic invariants. The only possible invariants have to be global. Classical invariants tell very little about this potentially interesting topological theory. It has long been suspected that there are other more subtle global invariants.

The turning point for symplectic geometry was in the early 1980s, when Gromov observed that much of the theory of holomorphic curves can be carried over to an almost complex manifold with a symplectic structure. In his historic paper [G], Gromov established the basic properties of the moduli space of pseudoholomorphic curves, particularly its compactness, which has become known as Gromov theory. Using this theory, Gromov proved many interesting theorems. After this work, Gromov's compactness was established in the context of the Uhlenbeck bubbling off [PW], [Wo]. Gradually, a remarkable resemblance between Donaldson gauge theory and Gromov symplectic theory emerged. Let us list two instances of this. (i) The noncompactness occurs for both cases by the Uhlenbeck bubbling off. (ii) Both theories study the bordism class of a finite-dimensional moduli space inside an infinite-dimensional manifold (the gauge equivalence classes of connections or $\mathrm{Map}(\Sigma, V)$), where Σ is a Riemann surface and V is a symplectic manifold). Perhaps the most famous example is Floer's homology theory for both gauge theory and symplectic geometry. To further explore the power of Gromov theory, it is natural to establish a Donaldson-

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