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# POSITIVE DIFFERENCE OPERATORS ON GENERAL MESHES

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**1. Introduction.** In this paper we consider maximum principles and resultant local estimates for linear positive difference operators on general meshes in Euclidean space. Our results extend these for cubic meshes in our previous paper [6] and also provide discrete analogues of the Aleksandrov-Bakelman maximum principle [1], [2] and the local estimates of Krylov and Safonov [5] and Trudinger [13] for linear elliptic partial differential operators.

In order to describe the results, we begin with some general definitions (following [9]). Let $E$ be an arbitrary set, which we shall call a *mesh*. A linear difference operator L acting on $\mathfrak{M}(E)$, the set of real mesh functions, is given by

$$Lu(x) = \sum_{y \in E} a(x, y)u(y) \tag{1.1}$$

for any mesh function $u$, where $a$ is a real-valued function on $E \times E$, which is nonvanishing for only a finite number of $y$ values for each $x \in E$. We call $L$ of *monotone type* if

$$a(x, y) \geqslant 0, \qquad \text{for all } (x, y) \in E \times E, \quad x \neq y, \tag{1.2}$$

and of *positive type* if, in addition,

$$c(x) \equiv \sum_{y} a(x, y) \leqslant 0. \tag{1.3}$$

If $D$ is a subset of $E$, then the *interior* of $D$, with respect to $L$, is defined by

$$D^o = \{x \in D \mid a(x, y) = 0, \quad \forall\, y \notin D\},$$

and the *boundary* of $D$, with respect to $L$ is defined by

$$D^b = D - D^o.$$

With these definitions, we conclude immediately a simple maximum principle. If $L$

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