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WAVE-TRACE INVARIANTS

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1. Introduction. Let X be a compact $(n+1)$-dimensional manifold and H a positive selfadjoint elliptic pseudodifferential operator of order 1 operating on the space of smooth half-densities, $C^\infty(X, \Omega^{1/2})$. The "wave trace" in our title refers to the sum over the eigenvalue, λ_k, of H:

$$e(t) = \sum e^{i\lambda_k t}.$$

By [Ch] and [DG], this is a tempered distribution in t with the following properties.

1. Let $\sigma(H)(x,\xi)$ be the leading symbol of H, and let Ξ be the Hamiltonian vector field on $T^*X - 0$ associated with $\sigma(H)$:

$$\Xi = \sum \frac{\partial}{\partial \xi_i}\sigma(H)\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i}\sigma(H)\frac{\partial}{\partial \xi_i}.$$

Then a necessary condition for a real number, T, to be in the singular support of $e(t)$ is that there exist a periodic trajectory, γ, of Ξ of period T (i.e., $\gamma(0) = \gamma(T)$).

2. Moreover, if γ is nondegenerate, it contributes to the wave trace a singularity of the form

$$e_\gamma(t) \sim \sum_{r=1}^{\infty} c_r (t - T + i0)^{-2+r} \log(t - T + i0); \tag{1.1}$$

and the coefficient, c_1, of the leading term in (1.1) is given by the formula

$$\frac{T_\gamma}{2\pi} i^{\sigma_\gamma} \left|\det(I - P_\gamma)\right|^{-1/2} \exp i \int_0^T \sigma_{\text{sub}}(H)(\gamma)\, dt, \tag{1.2}$$

where T_γ is the primitive period of γ (i.e., $\gamma(t) \neq \gamma(0)$ for $0 < t < T_\gamma$ and $\gamma(0) = \gamma(t)$ when $t = T_\gamma$), σ_γ is the Maslov index of γ, P_γ is the linearized Poincaré map about γ, and $\sigma_{\text{sub}}(H)$ is the subprincipal symbol of H (see [DG, §4].)

The wave-trace invariants that we will be concerned with in this article are the higher order terms in (1.1). In particular, one of our goals will be to verify a con-

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