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p-ADIC INTERPOLATION OF SQUARE ROOTS OF CENTRAL VALUES OF HECKE L-FUNCTIONS

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1. Introduction. Consider the L-series attached to powers of a given Hecke character of an imaginary quadratic field in which a prime p splits. It is known that suitable modifications of its special values can be p-adically interpolated. This gives rise to two-variable p-adic L-functions (see [13], [21], [3]). The modified central values of these complex L-series are known to be squares in a fixed number field. In [16], Koblitz conjectured that there exist appropriate choices of square roots of the central values, independent of the prime p, which can also be interpolated. In this article we will prove Koblitz's conjecture for a certain family of Hecke characters.

To do this, we use explicit formulas found by Rodriguez-Villegas and Zagier (Theorem 2.1) that give a natural choice of the square roots in question as values of certain half-integral weight theta-series at CM-points. We follow the ideas of Katz [13] and Serre [25]; roughly speaking, we first interpolate p-adically the sequence of theta-series as modular forms, and we then compose the interpolating function with evaluation at the CM-point. Of course, we need to make sense of this p-adic evaluation. We immediately encounter an obstacle: there is no complete theory for half-integral weight p-adic modular forms available yet. We therefore reduce our work to the integral-weight case, and make use of the existing theory.

Needless to say, the subject of half-integral weight p-adic modular forms is interesting by itself. There is ongoing work on this topic, initiated by Koblitz in [16], and pursued by Jochnowitz [8], [9], Stevens [34], and the author [32]. We intend to revisit it in a future paper.

We need to introduce some notation. Let $K = \mathbb{Q}(\sqrt{-l})$ for l a prime, $l \equiv 3 \pmod 4$, and let h_K be its class number. Let $L(\psi_k, s)$ be the L-function attached to $\psi_k = \psi^{2k-1}$, where ψ is a fixed Hecke character (see equation (1)). Fix an embedding $\overline{\mathbb{Q}} \hookrightarrow \mathbb{C}_p$, the completion of an algebraic closure of $\mathbb{Q}_p$.

Our main result is the following theorem.

THEOREM. *Let $p > 3$ be a rational prime not dividing h_K that splits completely in K and whose prime factors in K are principal. Then there is a continuous function $\mathscr{L}_p \colon \mathbb{Z}_p \to \mathbb{C}_p$ such that*

$$\frac{\mathscr{L}_p(k)}{c_{p,k}} = (\textit{Euler Factor})\frac{\sqrt{L(\psi_k, k)}}{c_{\infty,k}}$$

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