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PROPER HOLOMORPHIC MAPPINGS BETWEEN REAL ANALYTIC DOMAINS IN $\mathbf{C}^n$

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1. Introduction. Let $D_1 \Subset \mathbf{C}^n$ and $D_2 \Subset \mathbf{C}^n$ be two bounded domains with real analytic boundaries. Let f be a proper holomorphic mapping from D_1 to D_2 that can be extended smoothly up to $\bar{D}_1$. Baouendi-Rothschild [BR1] and Diederich-Fornæss [DF] showed that f extends holomorphically across a boundary point $p \in \partial D_1$ if the normal component of f has nonvanishing derivative in the normal direction at p (i.e., $(\partial f_\nu/\partial \nu)|_p \neq 0$). We remark that their theorems are purely of local character and are stronger than what we stated here. (See also closely related work by Lewy [Le], Pincuk [Pi], Webster [We], Diederich-Webster [DW], Bell [Be], Baouendi-Jacobwitz-Treves [BJT], Baouendi-Bell-Rothschild [BBR].) In particular, this is the case when both domains are pseudoconvex. Later in [BR2], Baouendi-Rothschild proved that if the normal component of f is not flat at p, then the condition that $\partial f_\nu/\partial \nu \neq 0$ holds automatically. More recently, in [BR3], it was proved that the Hopf lemma for the normal component of f holds at $p \in \partial D_1$ if $f(p) \in M_2$ is minimally convex in a certain sense. On the other hand, there have appeared a circle of papers studying the boundary-unique continuation problems for holomorphic mappings from the upper half-disk in the complex plane. (See [ABR], [BL], [Alx1], [HK], [BR5], [BR6], [Alx2], [HKMP].)

In this paper, we first study the unique continuation property for the normal component of f at p in case $f(p)$ is minimal but not minimally convex, where f is proper as defined above. Our result, together with the already established result of Baouendi-Rothschild in the minimally convex case, yields the following theorem.

THEOREM 1. *Let $D_1, D_2 \subset \mathbf{C}^n$ be bounded domains with M_1 and M_2 as part of their boundaries, respectively. Assume that M_1 and M_2 are real analytic minimal hypersurfaces and f is a proper holomorphic mapping from D_1 to D_2, that is, C^∞ smooth up to M_1 and maps M_1 into M_2. Then the normal component of f is not flat at any point of M_1.*

As is known, Theorem 1, together with the results in [BR2] and [BR3], enables one to restate the results in [BR1], [DF] (see also the previous work mentioned above) in the following form.

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